Bidding Games on Graphs

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Joint work with Guy Avni (University of Haifa) et al.

Graph Games

Theory:

-connections to logic and automata
-fundamental complexity questions

Applications:

-well-formedness (realizability, compatibility) of open specifications
-reasoning about multi-agent systems (ATL)
-sequential synthesis, discrete-event control, and AI planning

Graph Games

finite set Q of labeled nodes	=	system states
directed edges	=	state transitions
players Max, Min	=	decision agents
outcome $w \in Q^* \cup Q^\omega$	=	finite or infinite path
objective $\varphi: Q^\omega \to \text{Bool} \cup \text{Real}$	=	qualitative or quantitative specification

Objectives

Reachability:

given target $T \subseteq Q$,

$\varphi(w) = \}\ T(w) = \exists i.\ w(i) \in T$

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Parity:

given priority labels $p: Q \to Nat$,
$\varphi(w) = 1$ if $\max \{\ p : \exists^{\infty} i.\ p = p(w(i))\ \}$ is odd
$\varphi(w) = 0$ else

1
2
3
4
2
3
2
3
4
2
3
2
3
4
⋮

Objectives

1
2
3
4
2
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4
⋮

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Mean payoff:

given payoff labels $p: Q \to Nat$,
$\varphi(w) = \liminf_{n\to\infty} 1/n \cdot \sum_{0\leq i\leq n} p(w(i))$

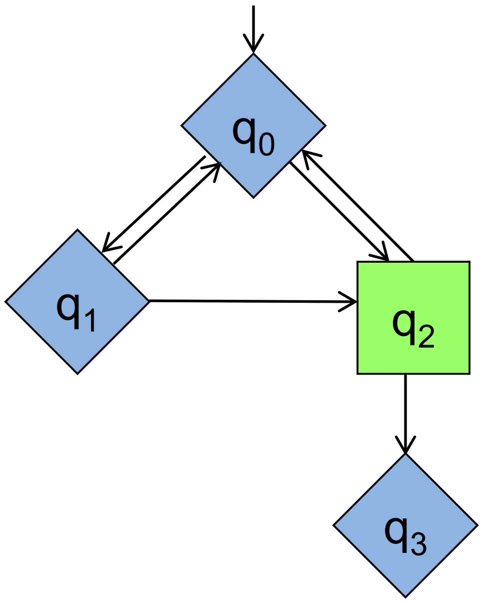
1	1
-1	0
1	1/3
2	3/4
-1	2/5
⋮	⋮

largest eventual lower bound

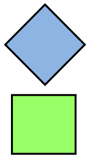
How to Generate a Path

1. Turn-based
2. Stochastic
3. Concurrent
4. Bidding

Turn-based Game Graph



Asynchronous interaction of players:



player Max chooses outgoing edge

player Min chooses outgoing edge

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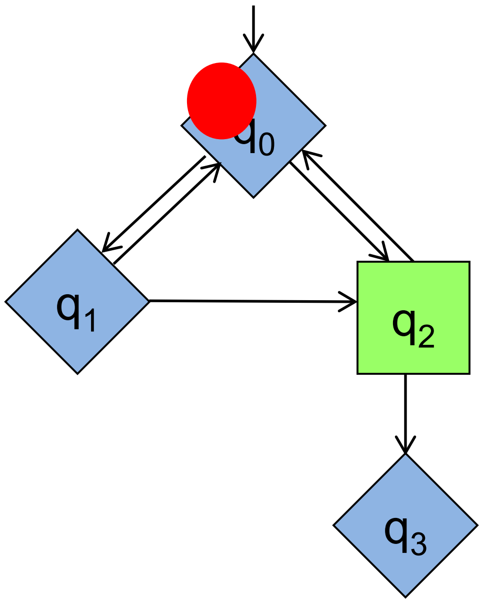

q_0

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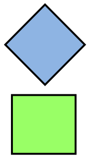

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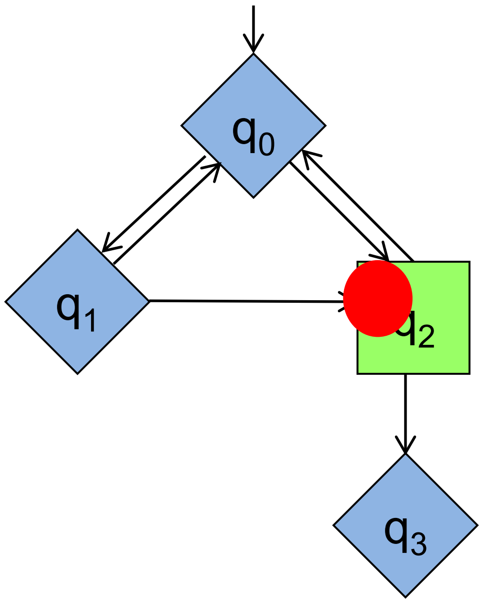

q_0

q_2

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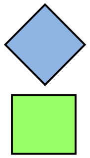

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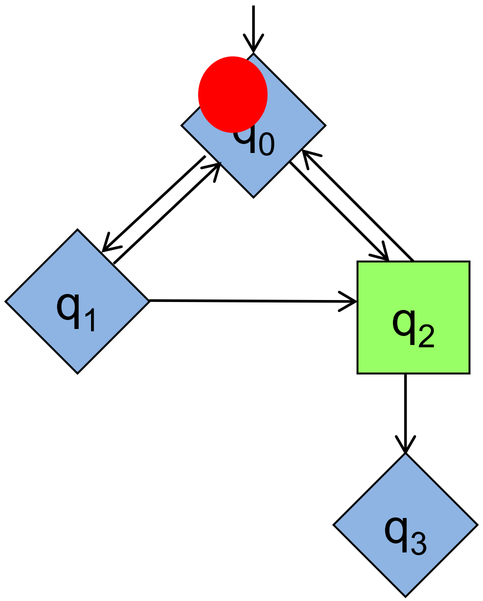

q_0

q_2

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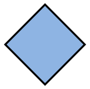

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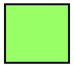

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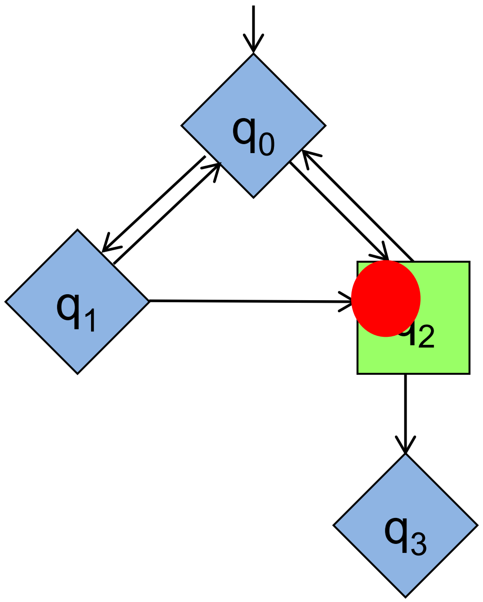

q_0
q_2
q_0
q_2

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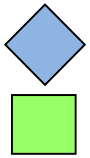

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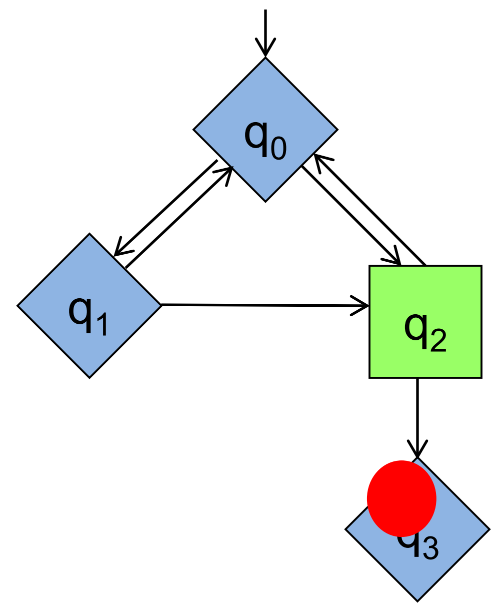

q_0

q_2

q_0

q_2

q_3

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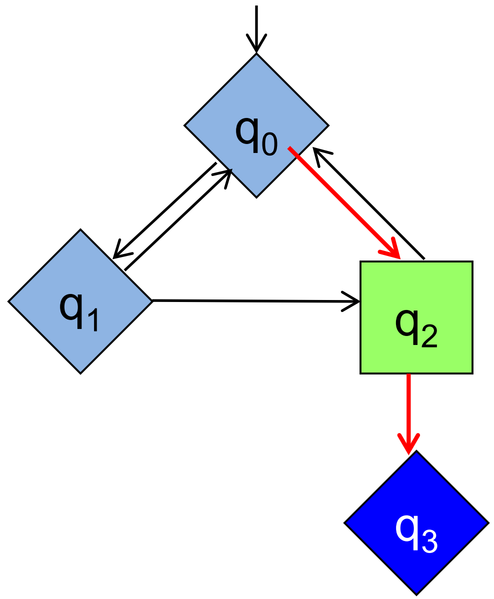


$x(q_0) = q_2$
$y(q_2) = q_3$

q_0

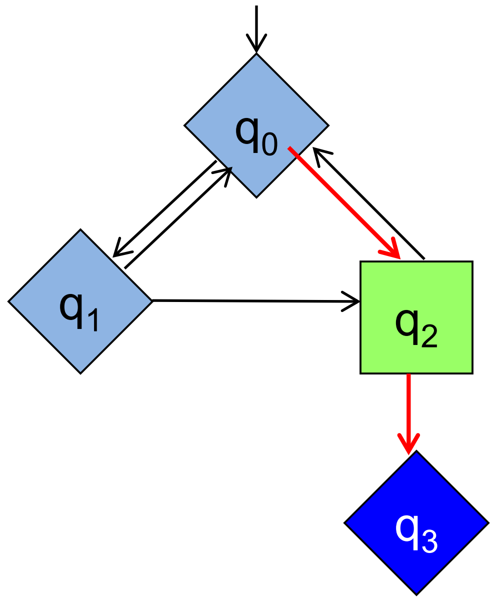
q_2

q_3

state q 2 Q
positional strategies x,y: Q ! Q
(x,y)@q: path in Q!

} q_3 ((x,y)@q_0) = 1

Turn-based Game Graph



$x(q_0) = q_2$
$y(q_0 q_2) = q_3$
$y(q_0 q_1 q_2) = q_0$

q_0

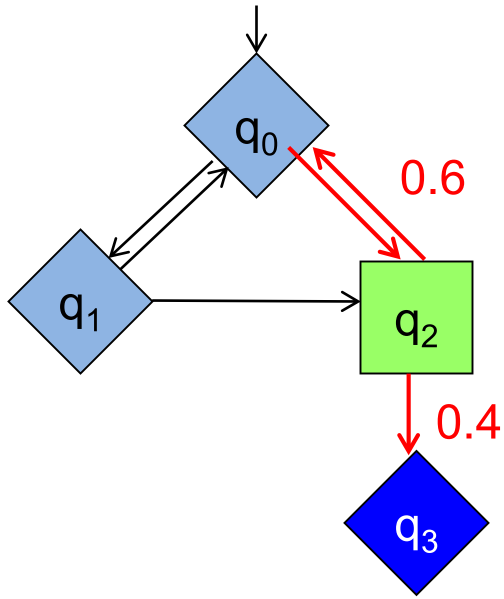
q_2

q_3

state q 2 Q
pure strategies x,y: Q^* ! Q
(x,y)@q: path in $Q^!$

} q_3 ((x,y)@q_0) = 1

Turn-based Game Graph



$x(q_0) = q_2$

$y(q_0q_2) = \{q_3\text{: } 0.4;\ q_0\text{: } 0.6\}$

state q 2 Q

mixed strategies x,y: Q^* ! D(Q)

(x,y)@q: probability space on Q!

Pr(} q_3) ((x,y)@q_0) = 1

Winning

Values at state q:

maxwin(q) = $\exists x.\ \forall y.\ \varphi((x,y)@q)$ for pure strategies, qualitative φ
= $\sup_x \inf_y \varphi((x,y)@q)$ for pure strategies, quantitative φ

maxval(q) = $\sup_x \inf_y \Pr(\varphi)\ ((x,y)@q)$ for qualitative φ
= $\sup_x \inf_y \mathrm{Exp}(\varphi)\ ((x,y)@q)$ for quantitative φ

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minwin(q) = $\exists y.\ \forall x.\ \neg\varphi((x,y)@q)$
minval(q) = $\inf_y \sup_x \Pr(\varphi)\ ((x,y)@q)$

Winning

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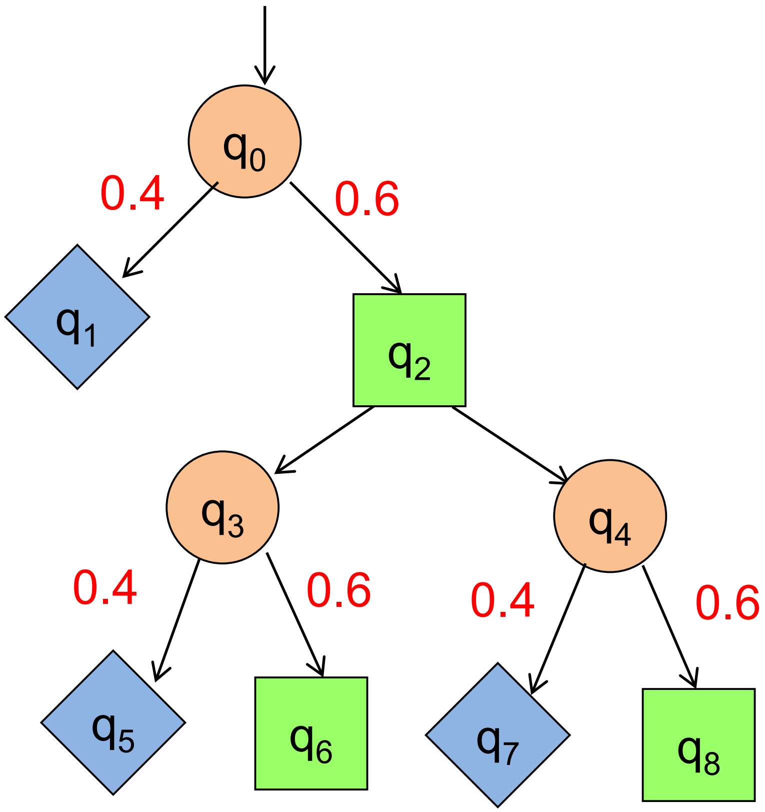
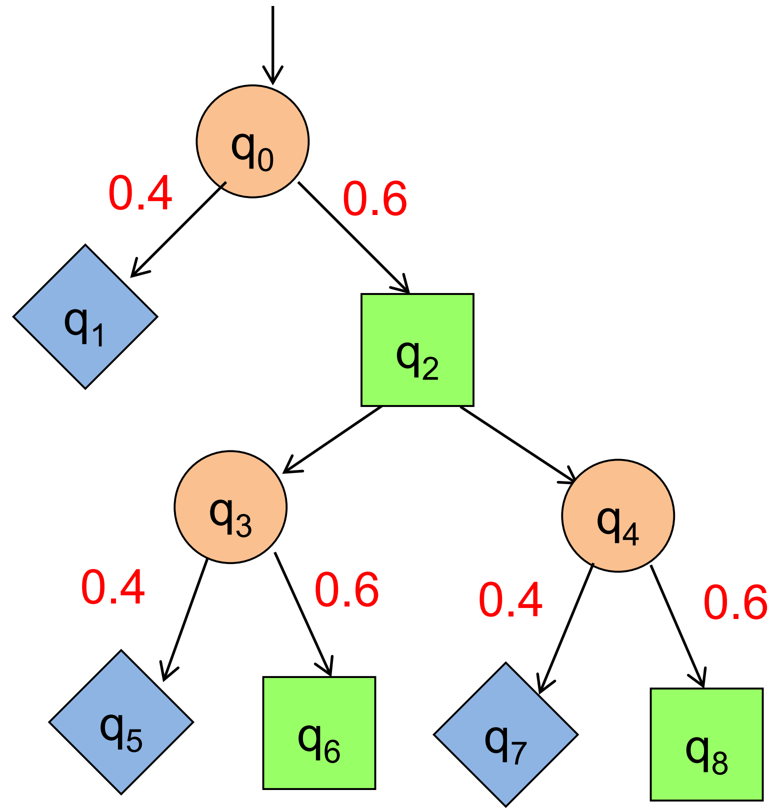
Determinacy:

$\forall q.$ maxwin(q) = 1 – minwin(q)
$\forall q.$ maxval(q) = minval(q) = val(q)

Stochastic Game Graph

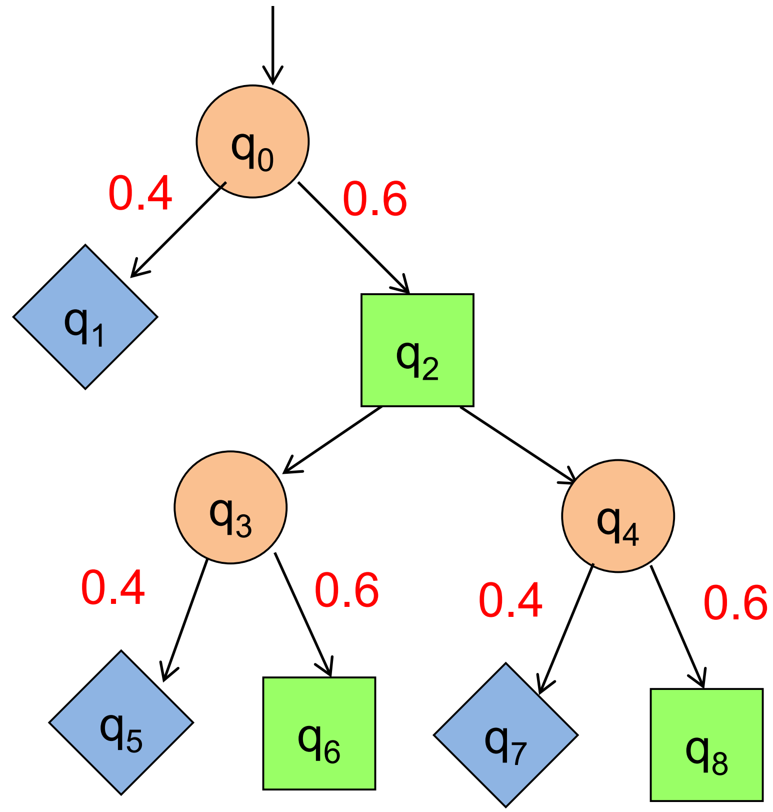


Stochastic Game Graph



Determined for positional strategies:
val in NP $\cap$ coNP for parity and mean-payoff.

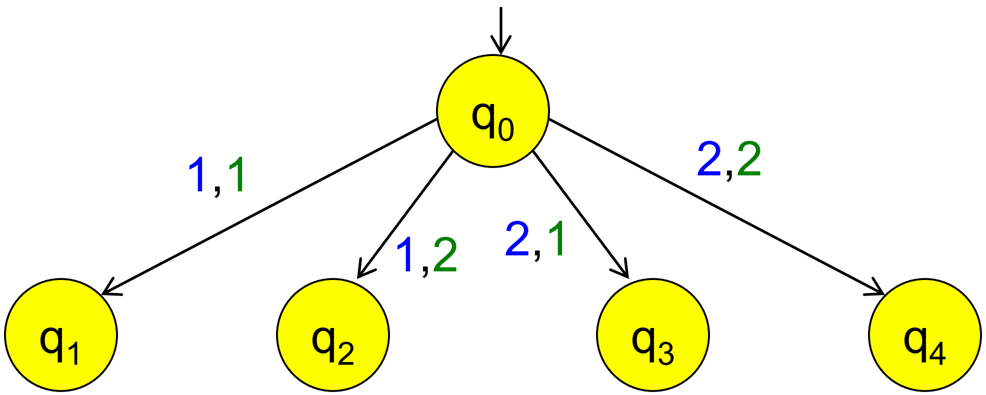
Stochastic Game Graph



Random-turn game.

Determined for positional strategies:
val in NP $\cap$ coNP for parity and mean-payoff.

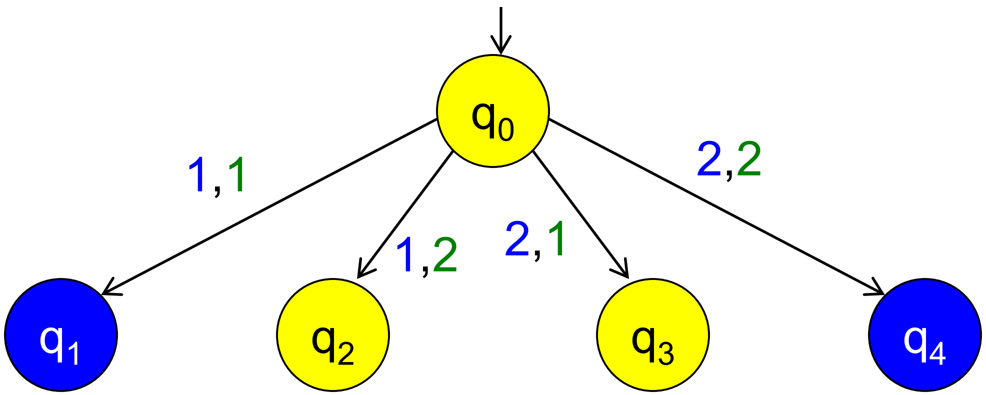
Concurrent Game Graph



Synchronous interaction of players:

-player Max moves {1,2}
-player Min moves {1,2}

Concurrent Game Graph



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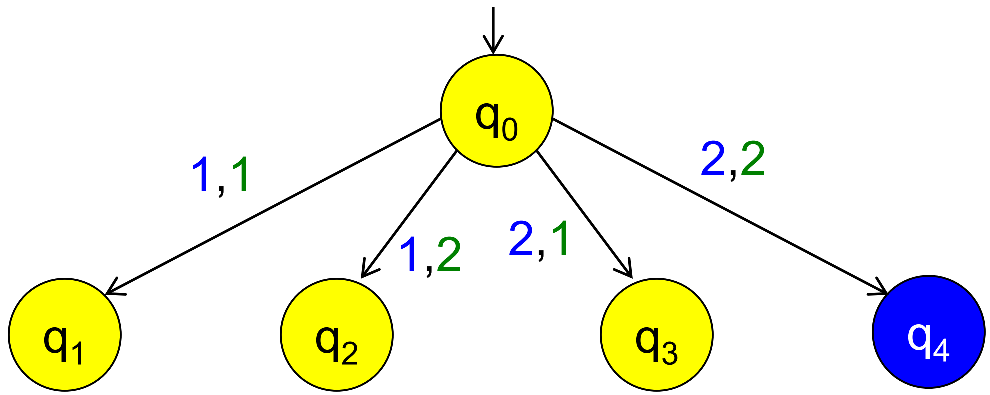
Not determined for pure strategies.

Concurrent Game Graph



state q 2 Q

strategies x,y: Q^* !   D(Moves)

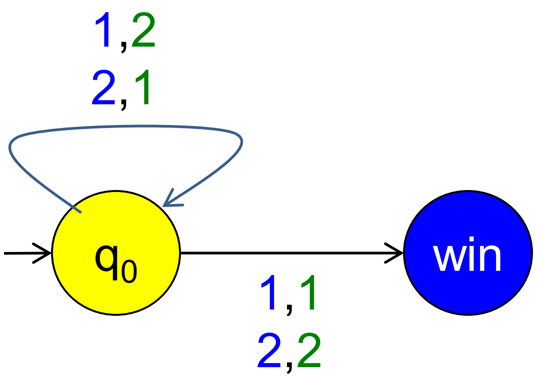
(x,y)@q: probability space on Q!

x(q_0) = 2

y(q_0) = {1: 0.4; 2: 0.6}

Pr(} q_4) ((x,y)@q_0) = 0.6

“Matching Pennies”



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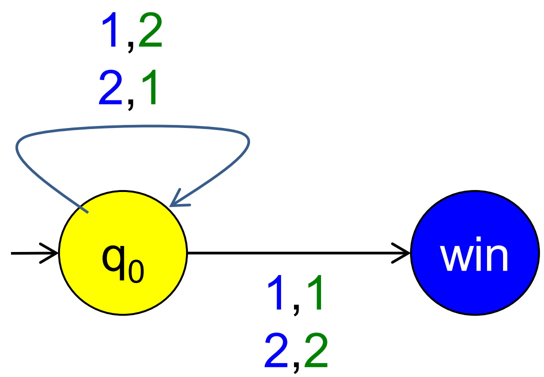
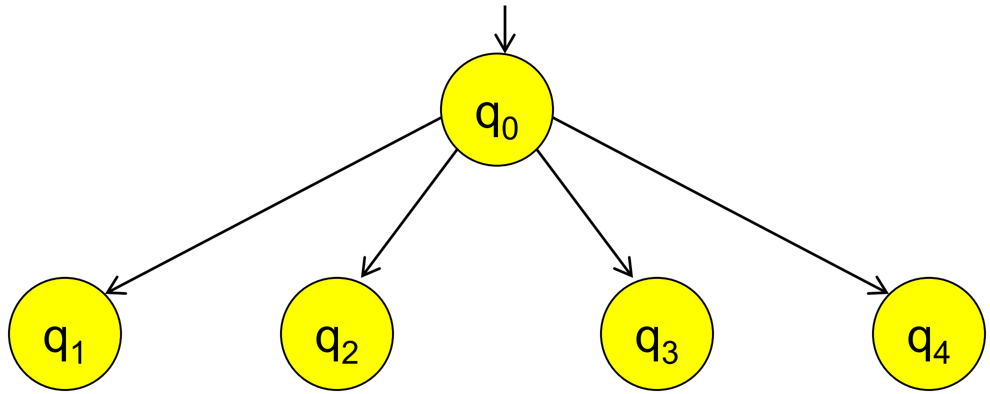


Max can use randomness to win:

maxwin(} win@q_0) = 0 for pure strategies
val(} win@q_0) = 1 for mixed strategies

Matrix Game Graph



player Min

q_0:

		1	2
player Max	1	q_1: 0.3 q_2: 0.2 q_3: 0.5 q_4:	q_1: 0.1 q_2: 0.1 q_3: 0.5 q_4: 0.3
	2	q_1: q_2: 0.2 q_3: 0.1 q_4: 0.7	q_1: 1.0 q_2: q_3: q_4:

Matrix game at each node.

Mixed determinacy.

Bidding Games on Graphs

Each player has a budget.
At each node, each player bids part of their budget.
The winning player chooses the outgoing edge.

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Combination of graph games and algorithmic game theory.

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Technical challenge: infinitely many possible moves
-budgets and bids are real numbers
-strategy: $(Q \times \text{Real} \times \text{Real})^* \to D(\text{Real}) \times D(Q)$

Bidding Modes

Current budgets: Bmax, Bmin
Current bids: maxbid, minbid; case maxbid > minbid

Richman bidding (David R. Richman 1980s):

New Max budget:	Bmax – maxbid
New Min budget:	Bmin + maxbid

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“Poorman” bidding (Andrew J. Lazarus et al. 1990s):

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"Poorman" bidding (Andrew J. Lazarus et al. 1990s):

New Max budget:	Bmax – maxbid
New Min budget:	Bmin

All-pay bidding:

New Max budget:	Bmax – maxbid
New Min budget:	Bmin – minbid

Applications

-repeated, stateful auctions (first-price; second-price similar)

-poorman: pay for services with limited capacity;
for being scheduled or routed (cloud, network, blockchain)

-all-pay: allocate bounded resources that are being consumed
(military reserves, replacements in team sports)

-Richman: decentralized arbitration between different providers

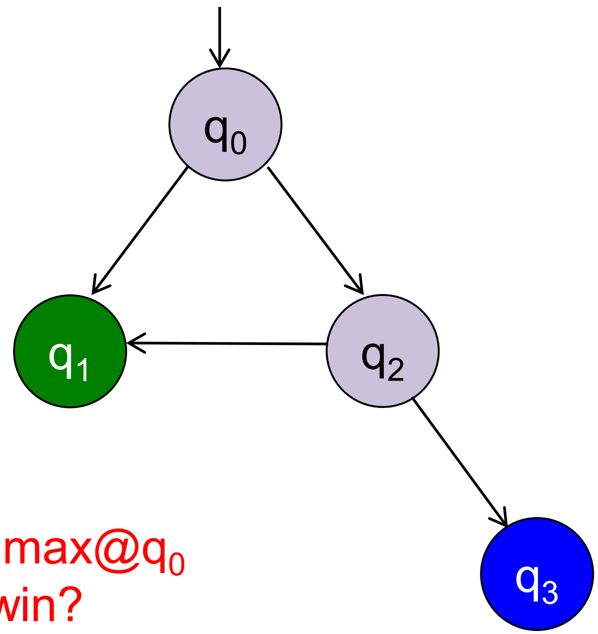
“Win Twice in a Row”

Player Max
objective: } q_3

Player Min
initial budget:
$Bmin@q_0 = 1$

How much initial budget $Bmax@q_0$
does player Max need to win?

Which bidding mode does player Max prefer?



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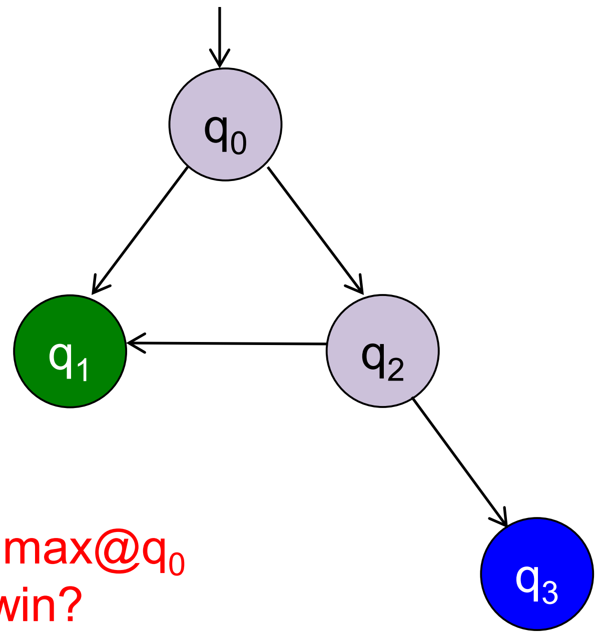

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Richman: 3

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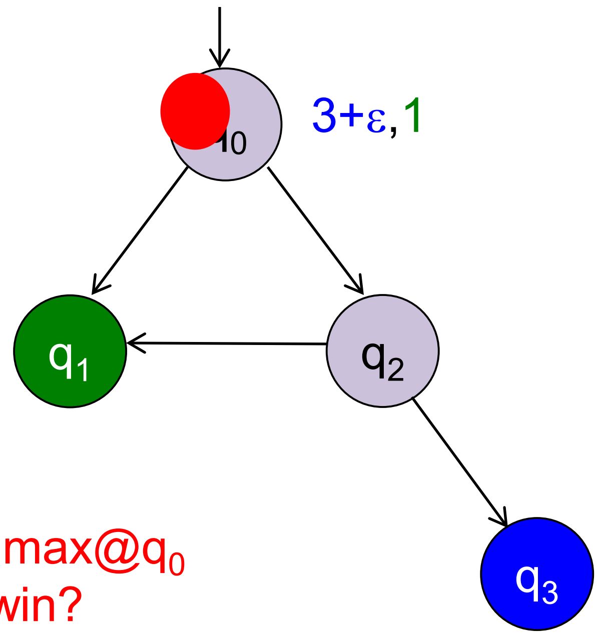

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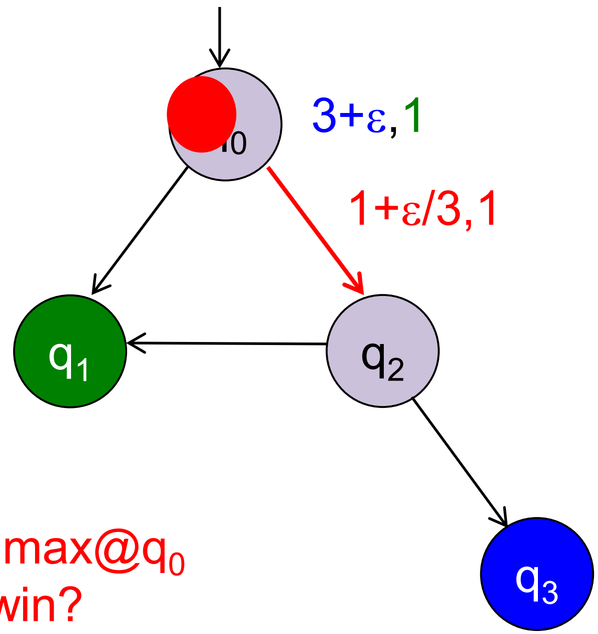

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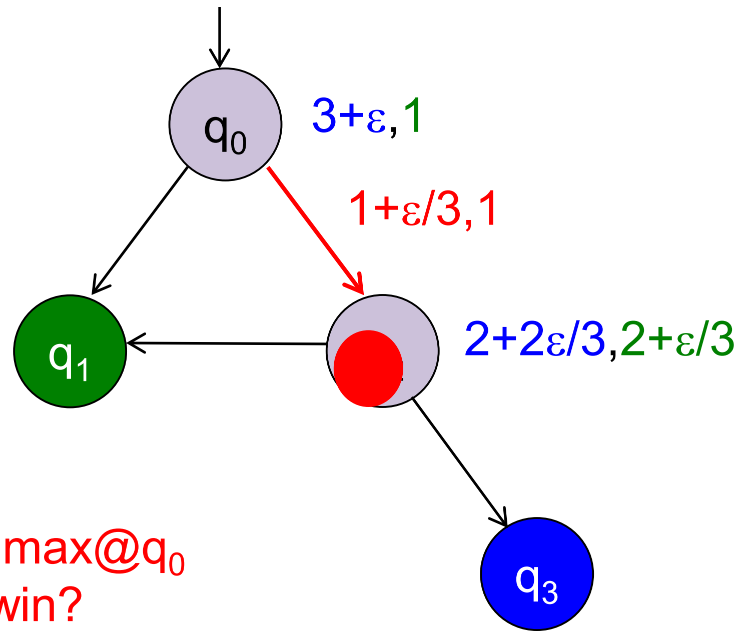

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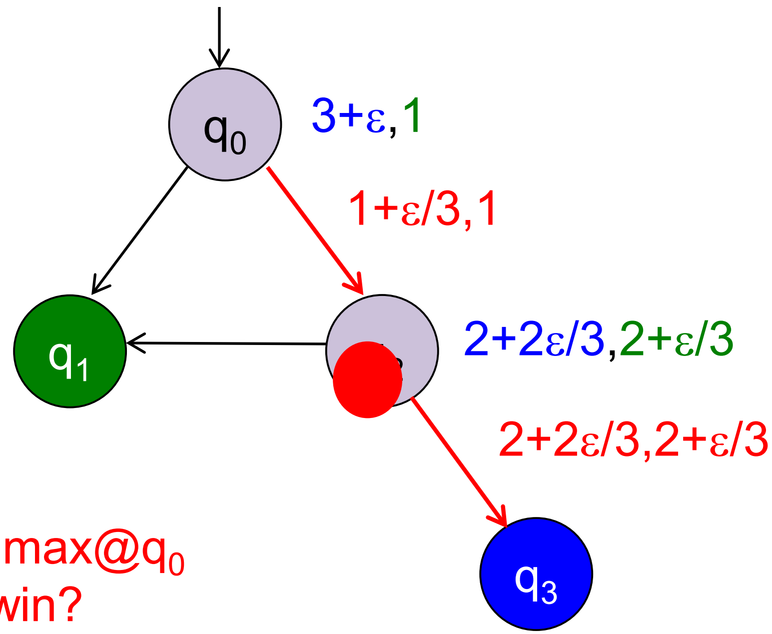

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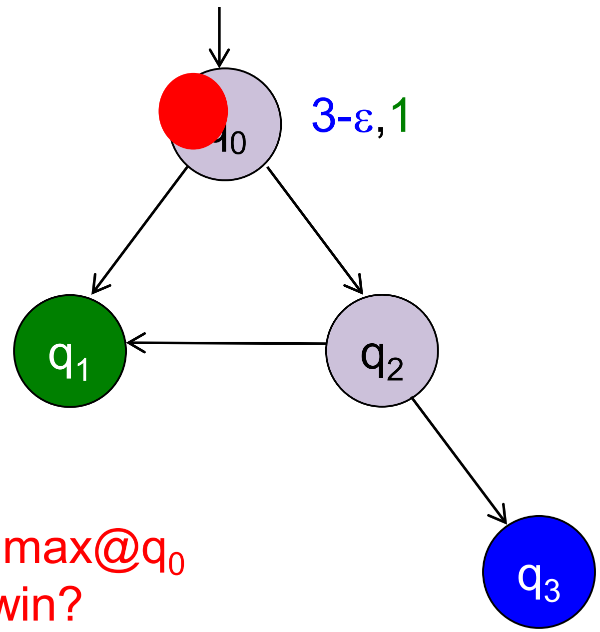

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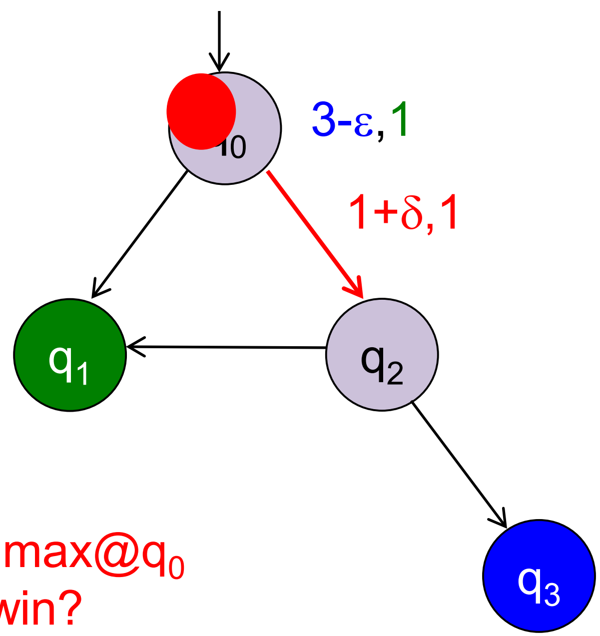

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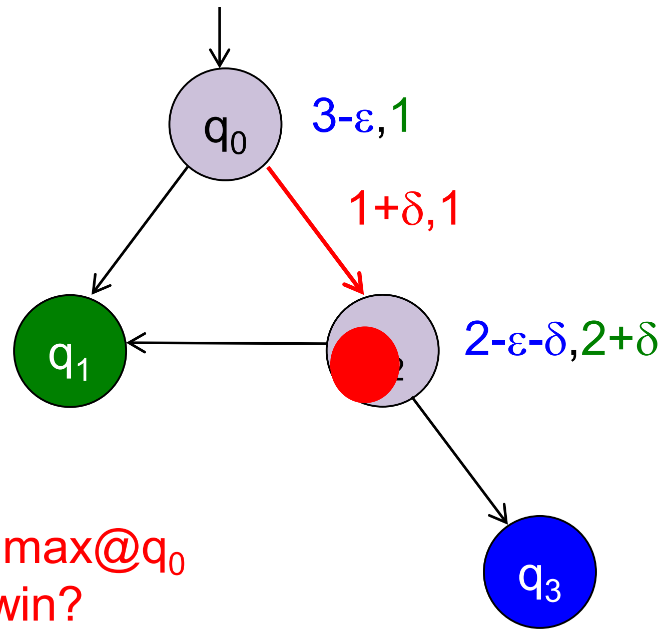

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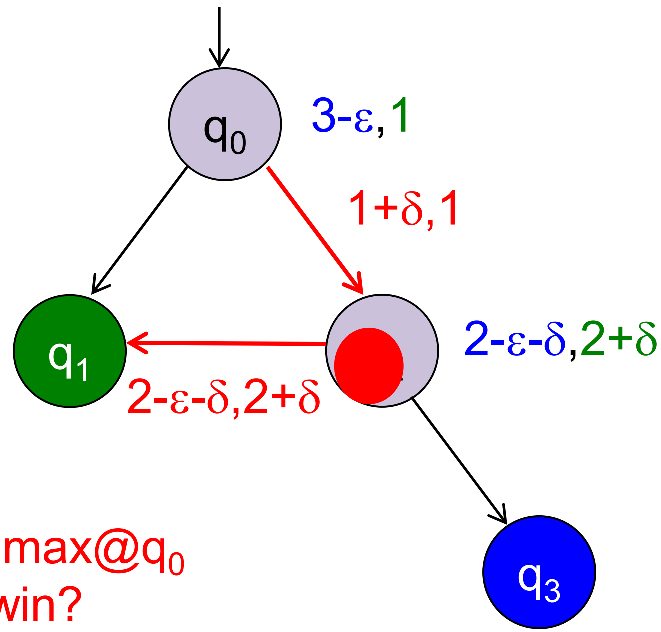

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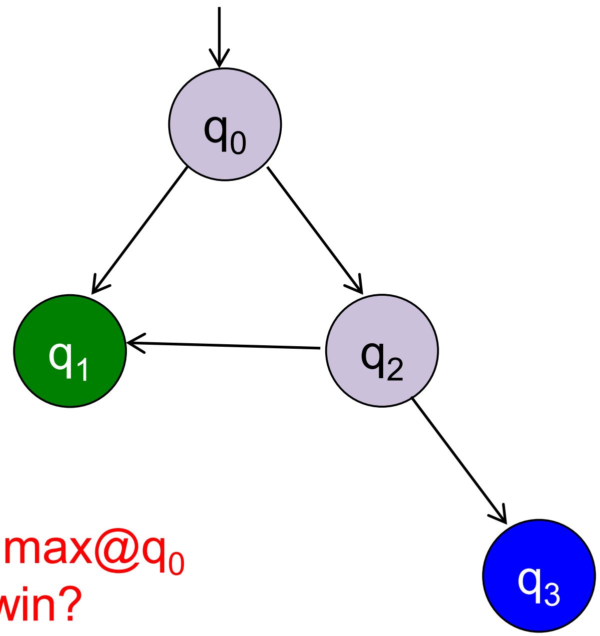

How much initial budget Bmax@q_0
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Richman:	3
Poorman:	2

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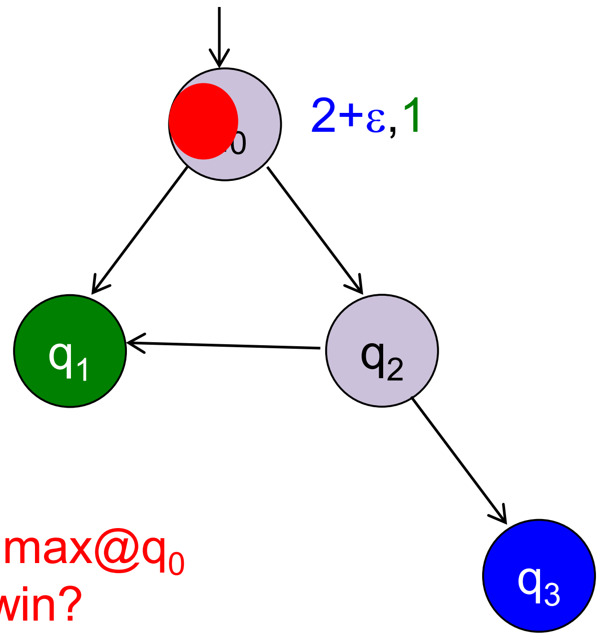

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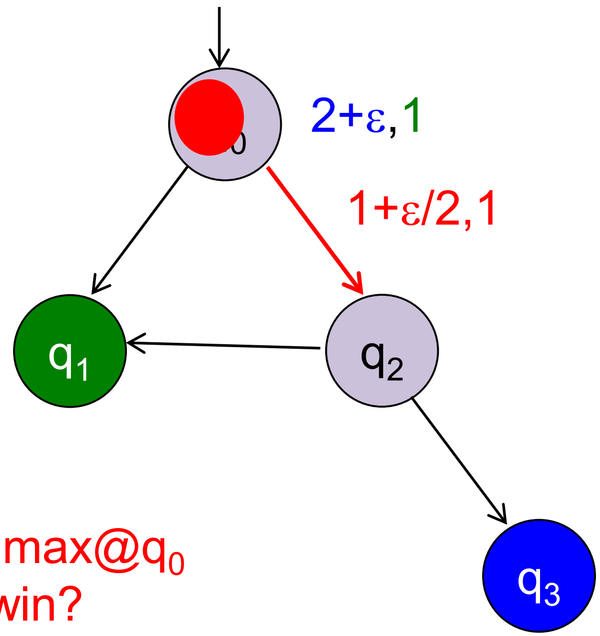

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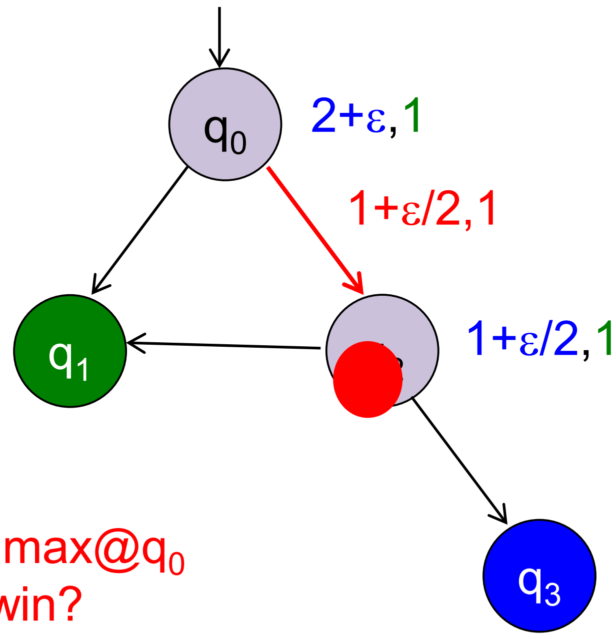

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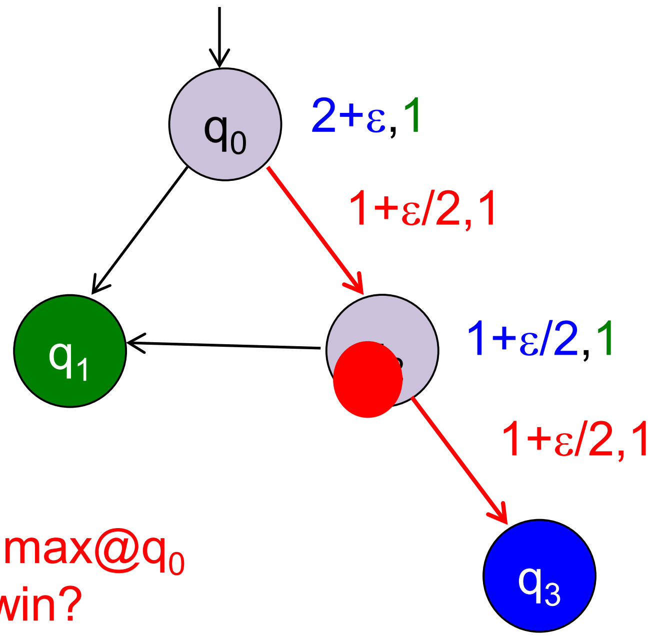

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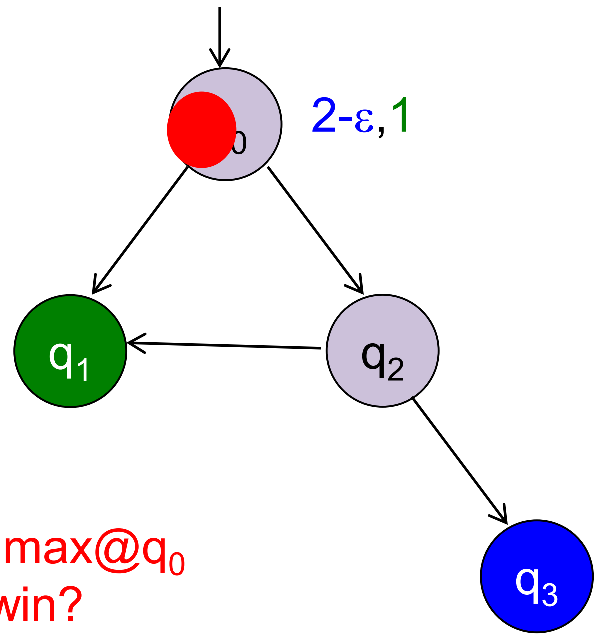
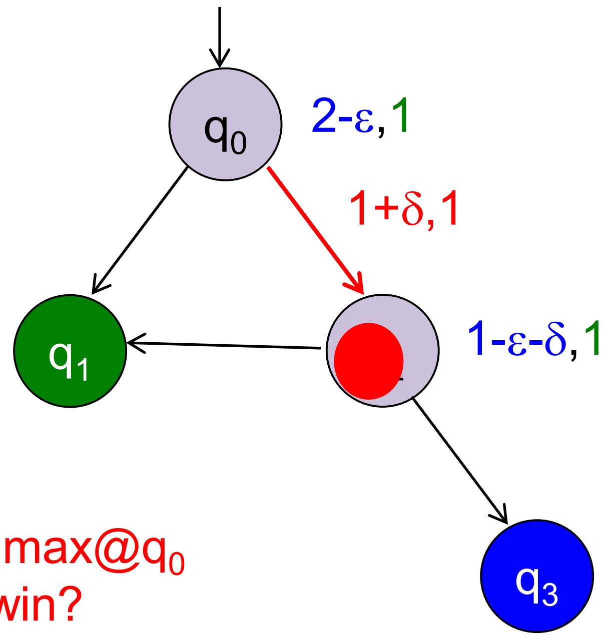


Richman: 3
Poorman: 2

“Win Twice in a Row”

Player Max
objective: } q_3

Player Min
initial budget:
Bmin@q_0 = 1



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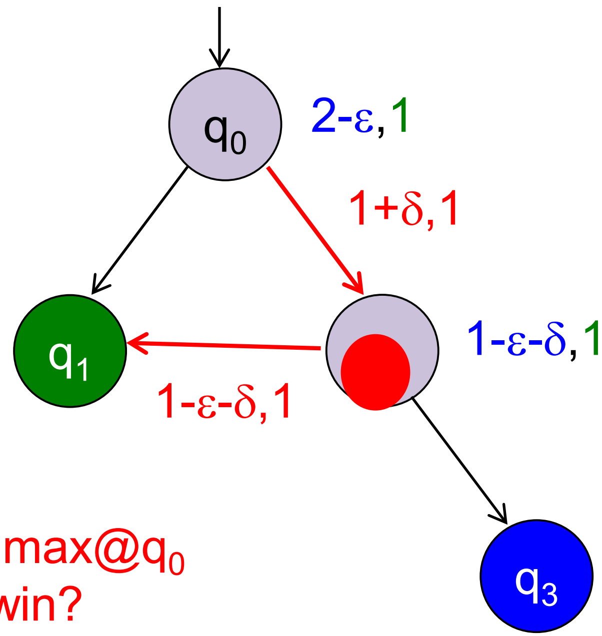

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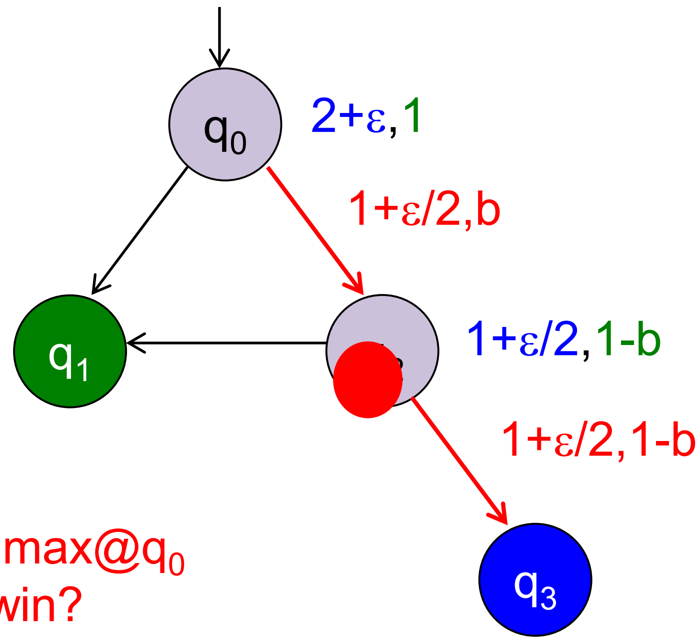

Richman:	3
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All-pay:	$Bmax@q_0 > 2$: Max wins

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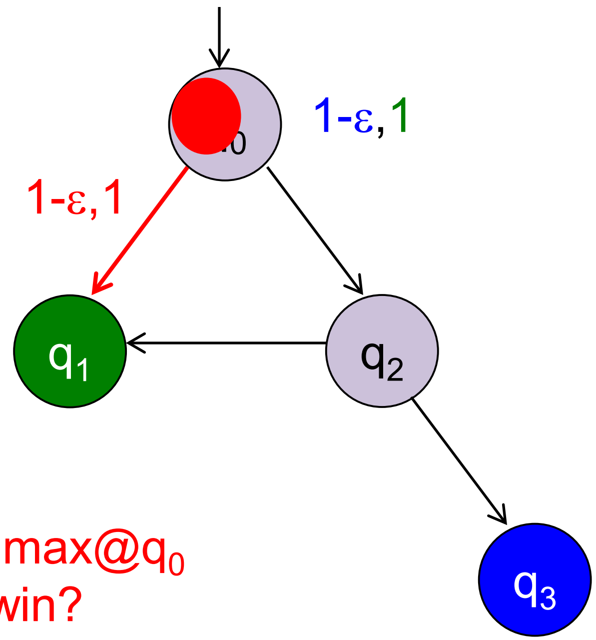

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Richman:	3
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	Bmax@q_0 < 1: Min wins

"Win Twice in a Row"

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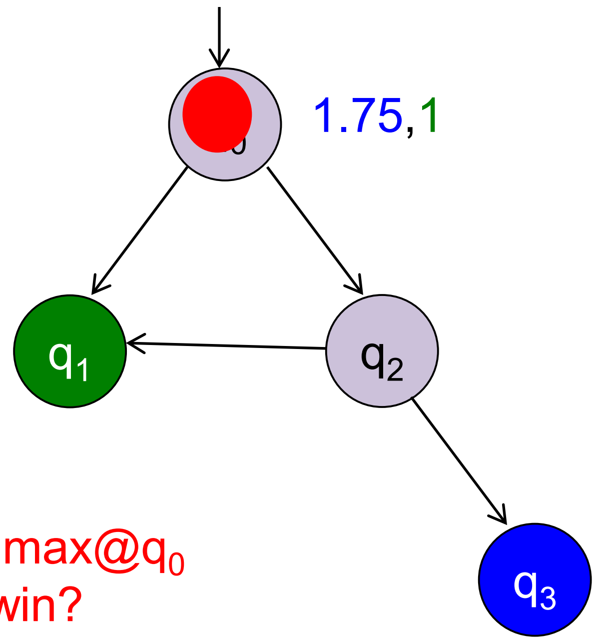

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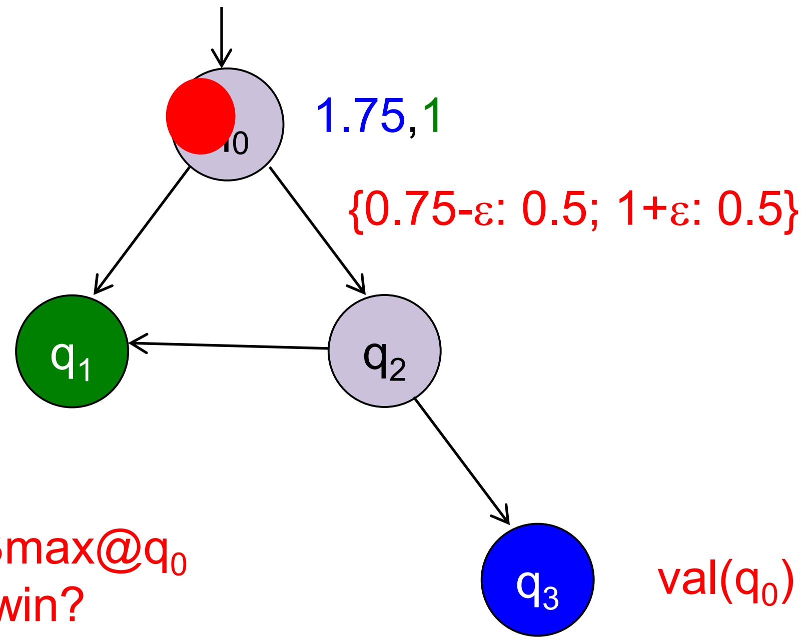

val(q_0) = 0.5

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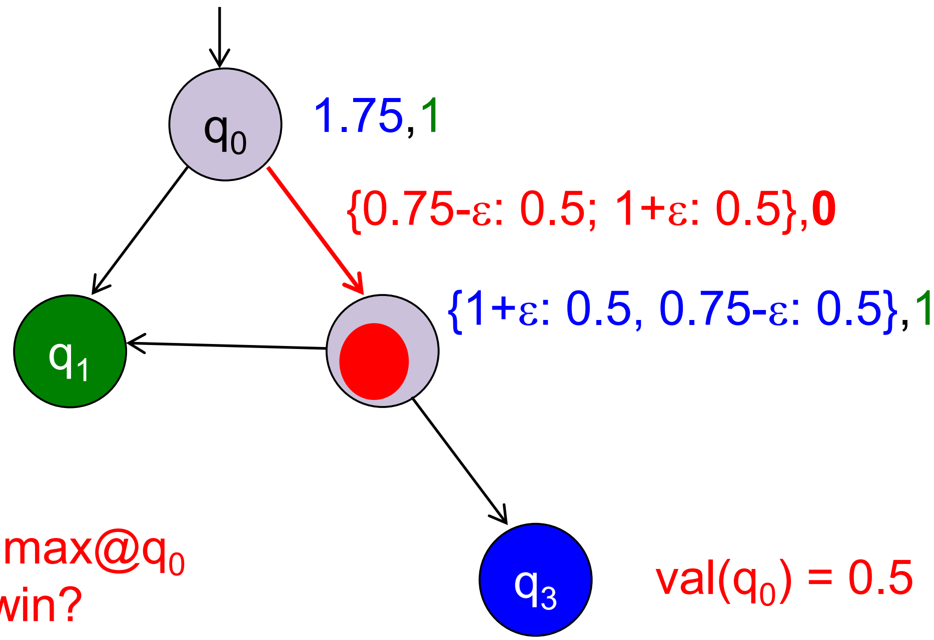

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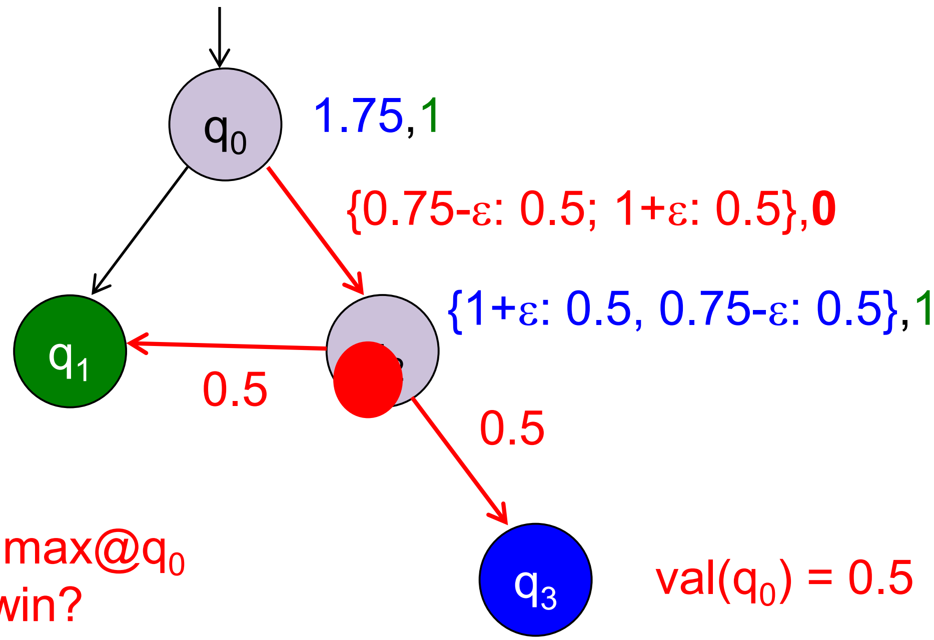

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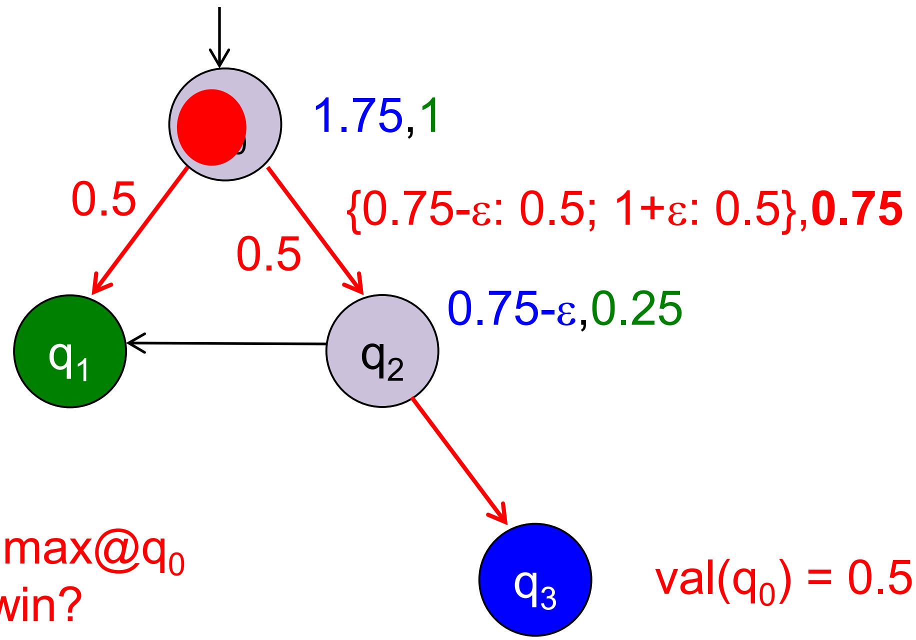

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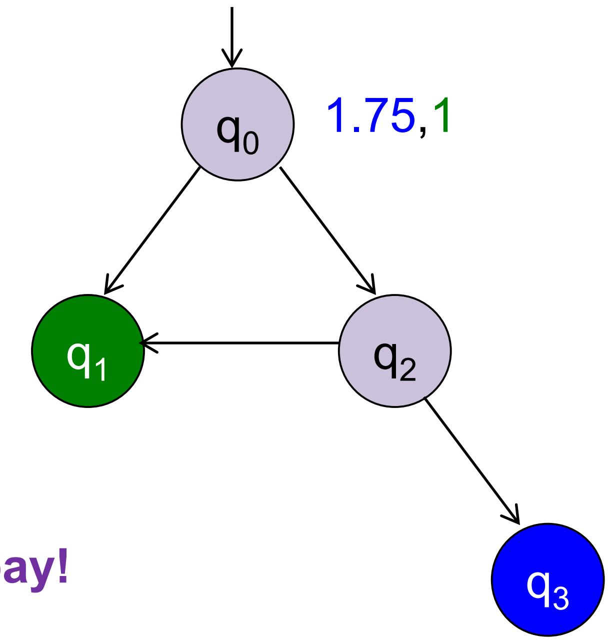
Player Max prefers all-pay!



1.75,1

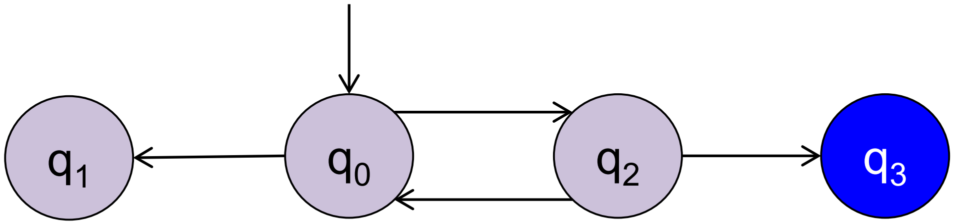
val(q_0) = 0.5

Thm [Avni, Ibsen-Jensen, Tkadlec]:
With all-pay bidding, for all n $\in$ Nat,
if Bmax@q_0 $\in$ [1+1/(n+1), 1+1/n], then val(} q_3@q_0)=1/(n+1).

Richman and Poorman

1. Reachability
2. Parity
3. Mean-payoff

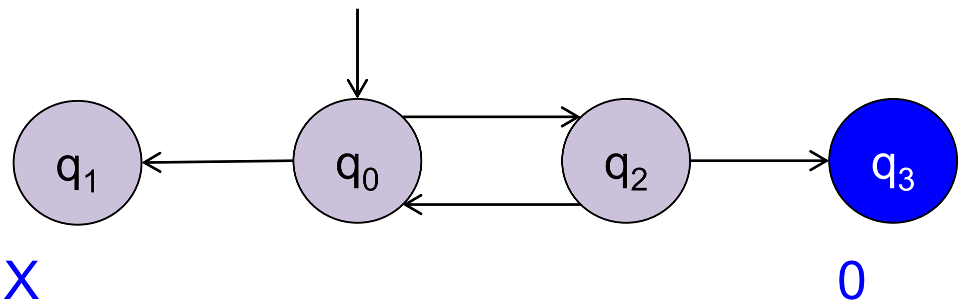
Richman Reachability



How much initial budget Bmax@q_0 does player Max need to win } q_3 ?

W.l.o.g. the sum of the initial budgets of players 1 and 2 is 1.
Richman invariant: Bmax + Bmin = 1.

Richman Reachability



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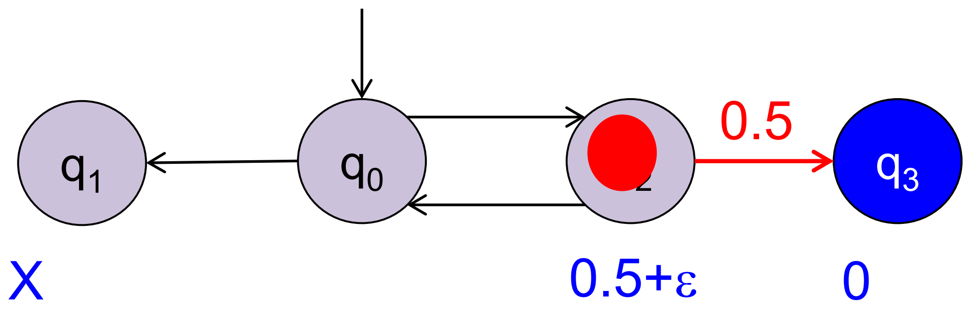

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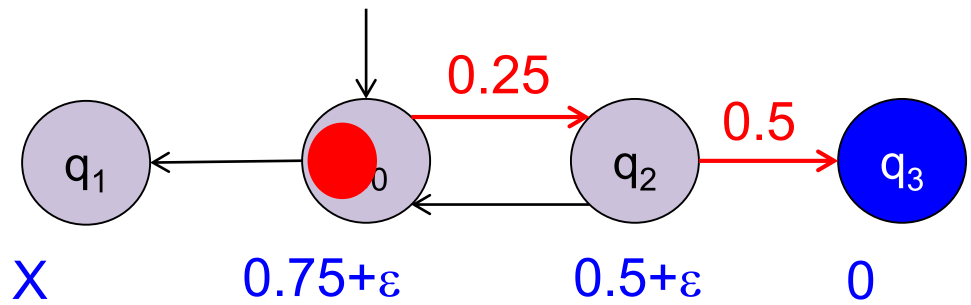

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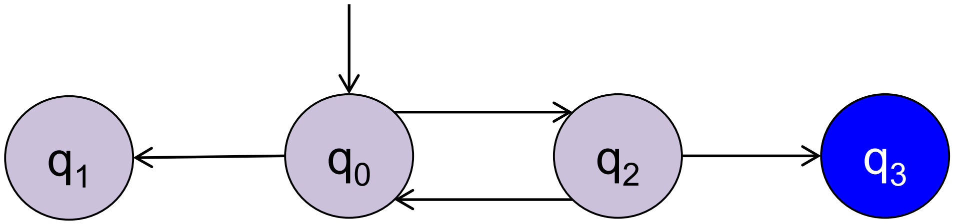

Threshold budget ratio R@q:
player Max wins if Bmax > R; else player Min wins.

Thm [Lazarus et al.]: threshold budget ratios exist for reachability.

What is the threshold budget ratio for player Max to win } q_3 ?

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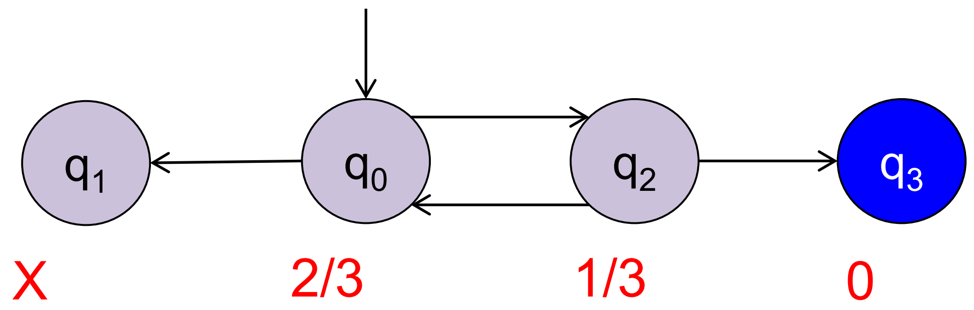

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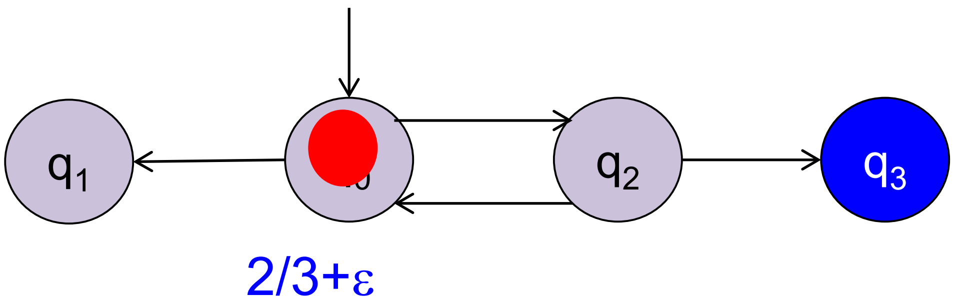

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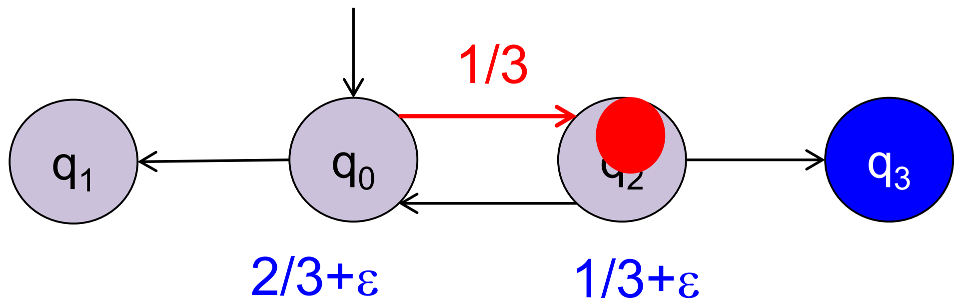

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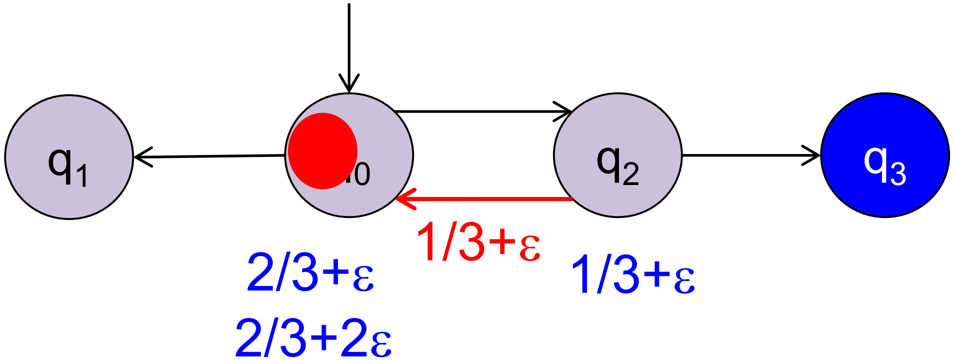

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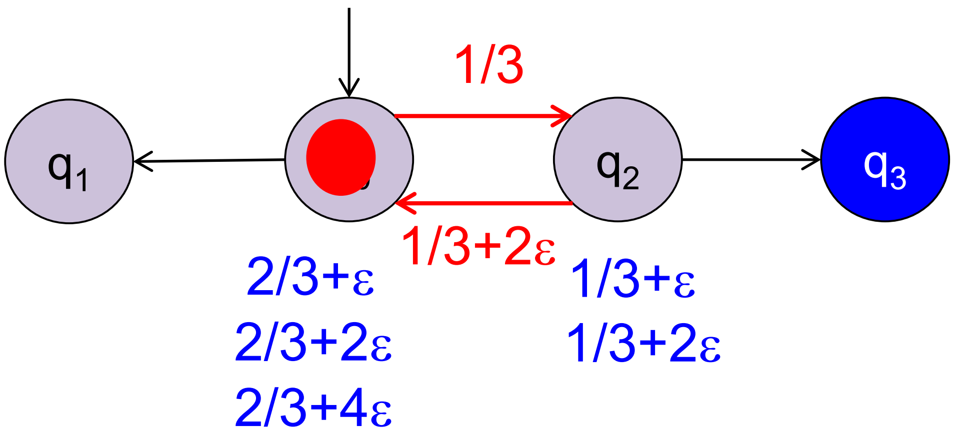

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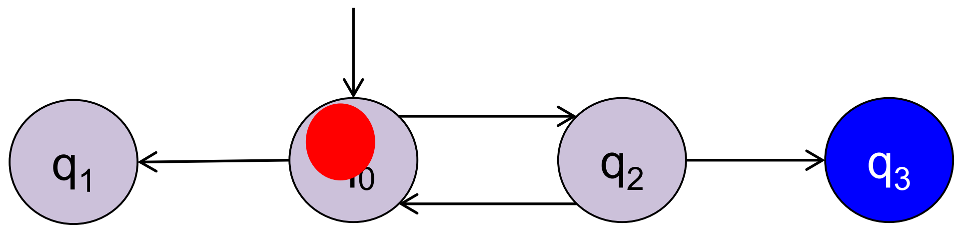

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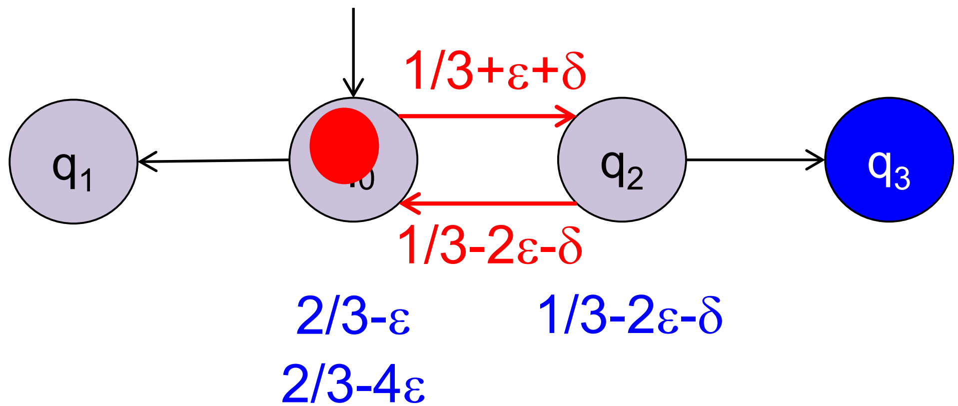

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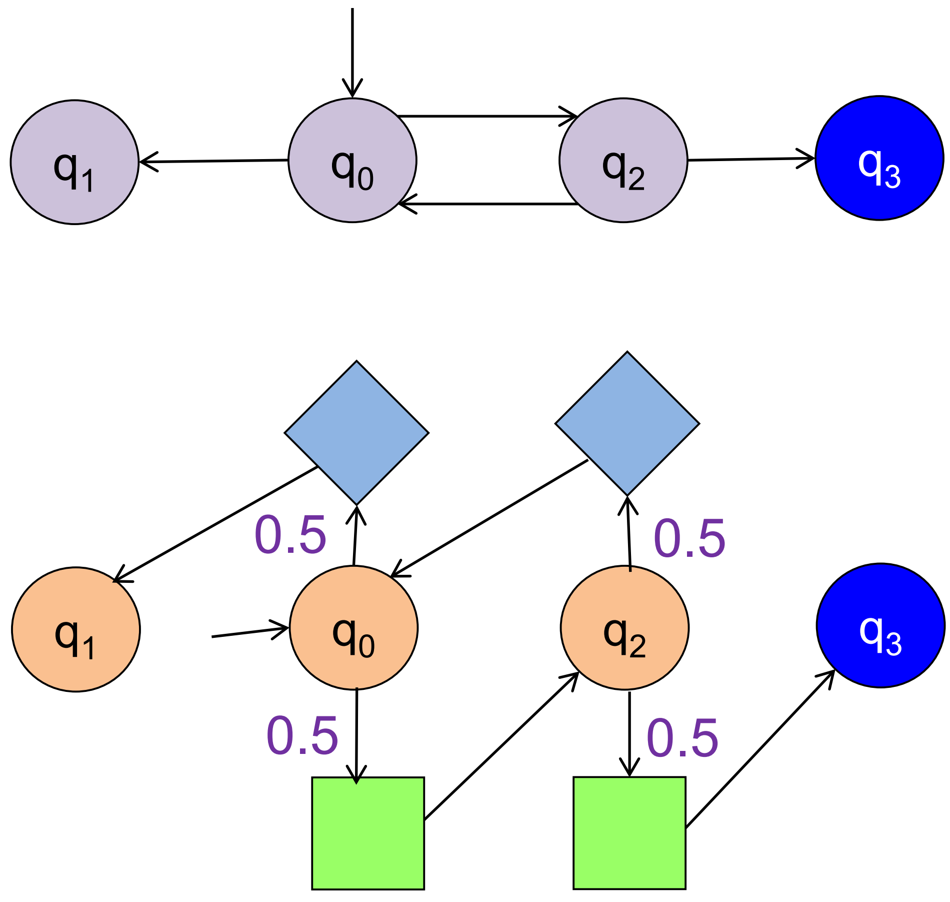
Equivalence with Uniform Random-Turn Games



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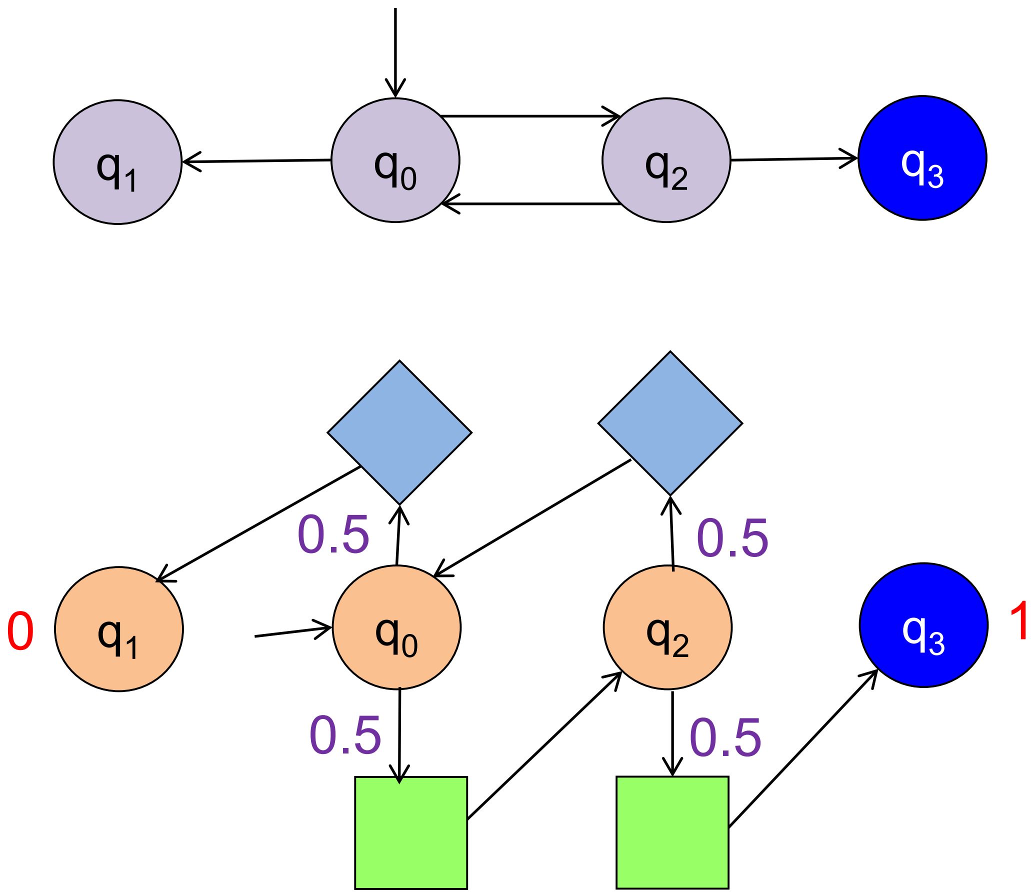

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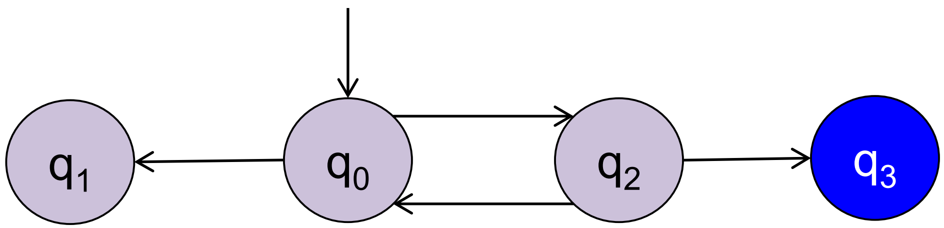

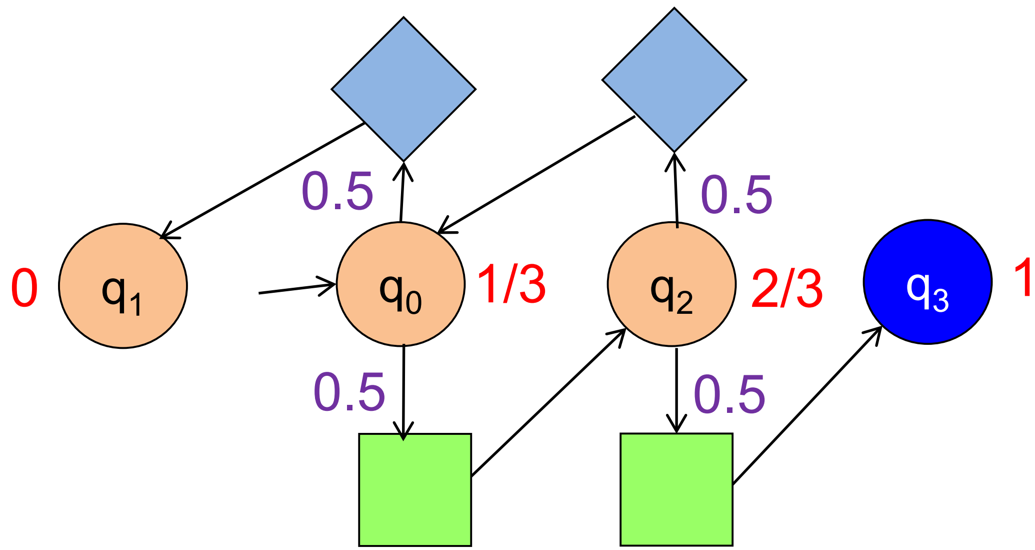

val(q) = 0.5 · (val(q-)+val(q+))

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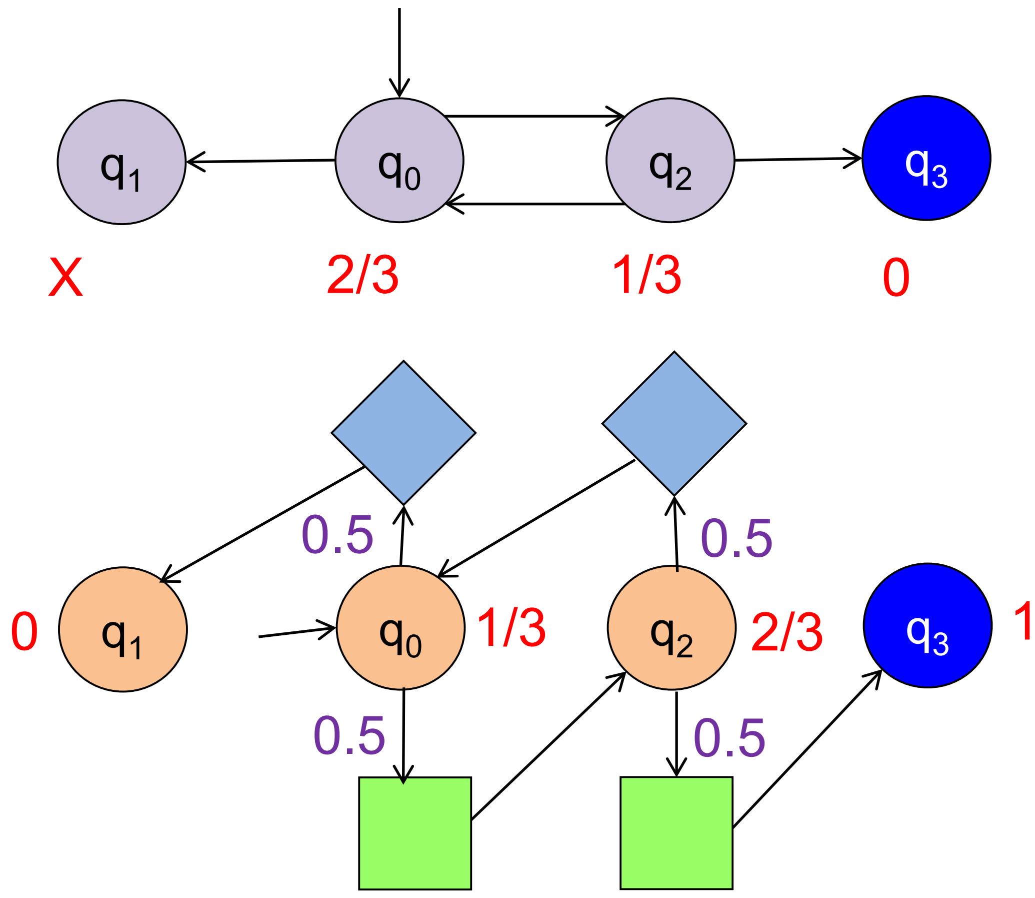

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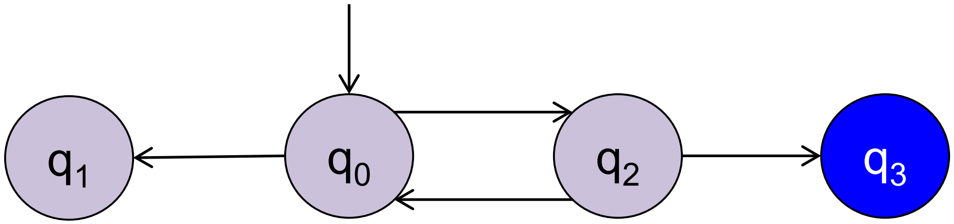
Poorman Reachability



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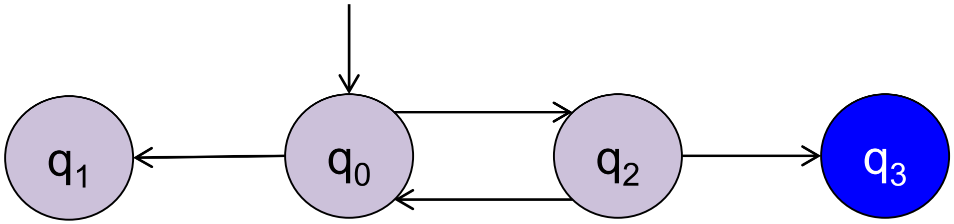

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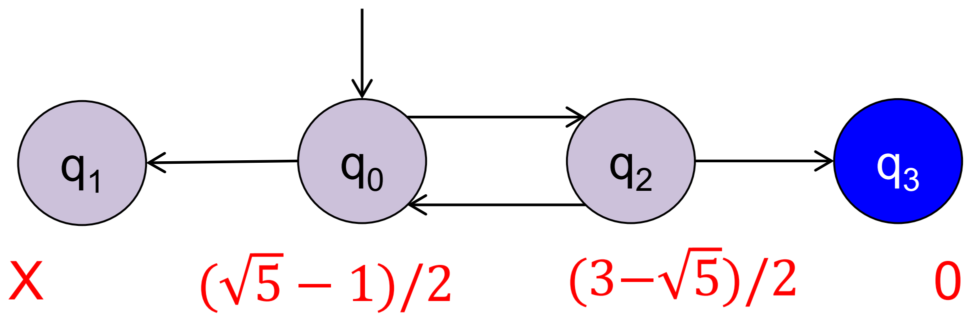

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No equivalence with random-turn games.

Richman and Poorman Parity

Richman and poorman (indeed, “taxman”) parity is no harder than reachability, because in each bottom strongly connected component of the game graph, the largest priority (odd or even) determines the winner.

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poorman parity is in PSPACE (via the existential theory of the reals).

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Finding threshold budget ratios for Richman parity is in NP $\cap$ coNP (via the equivalence with random-turn games);
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Tight complexity bounds are open for both Richman (between P and simple stochastic games) and poorman (between P and the existential theory of the reals) reachability and parity.

Richman and Poorman Mean-Payoff

Main result:

Unlike for qualitative objectives, for mean-payoff objectives, an equivalence with random-turn games holds in both the Richman and the poorman case.

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Richman and Poorman Mean-Payoff

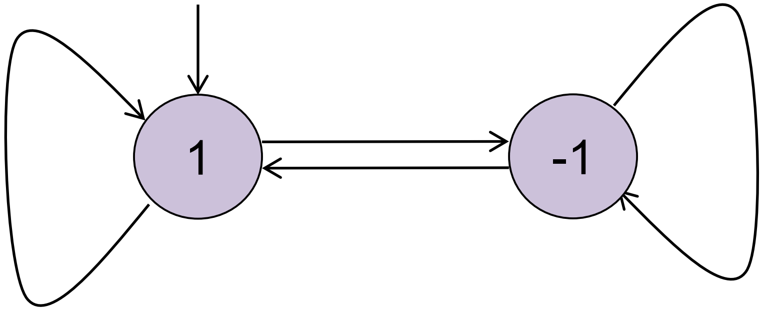
Main result:

Unlike for qualitative objectives, for mean-payoff objectives, an equivalence with random-turn games holds in both the Richman and the poorman case.

In the strongly-connected Richman case, the initial budget ratio does not matter, and the corresponding random-turn games are uniform.

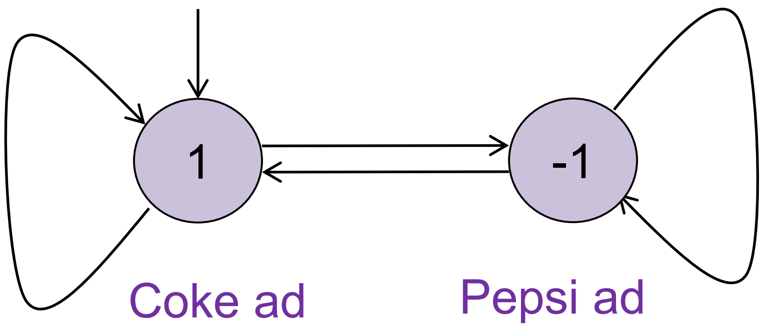
In the strongly-connected poorman case, the corresponding random-turn games are biased by the initial budget ratio.

"Bowtie"



Player Min tries to minimize mean payoff (always chooses right node).
Player Max tries to maximize mean payoff (always chooses left node).

"Bowtie"



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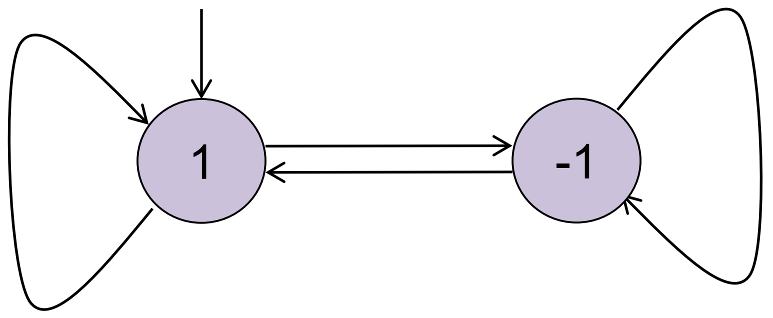
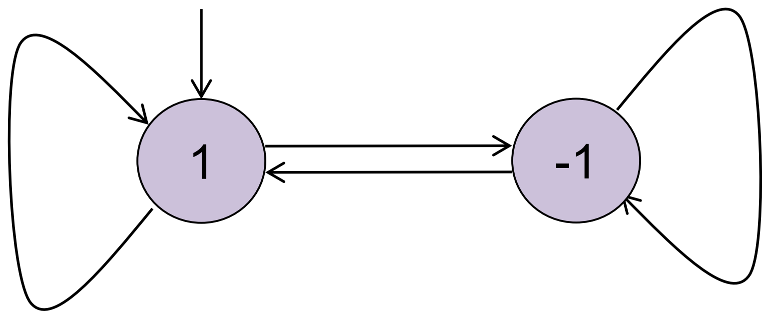


Player Min tries to minimize mean payoff (always chooses right node).
Player Max tries to maximize mean payoff (always chooses left node).

What is the threshold budget ratio for player Min to achieve val ≤ 0 ?

Which bidding modes do the players prefer?

Richman "Bowtie"



Value of strongly-connected game is 0 independent of initial budgets.

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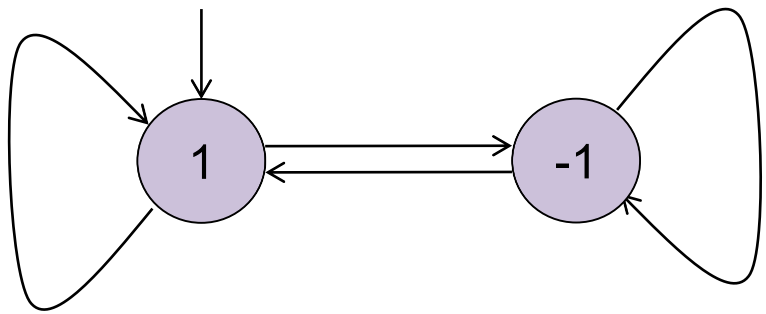

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Optimal strategy for player Min:
remember multiset M of winning Max bids so far;
bid largest member of M and remove it from M if winning;
if M is empty, bid 0.

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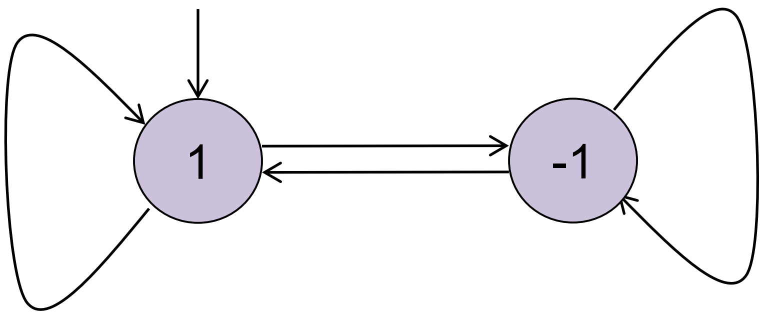

-1 ∅
+1 1/17
-1 ∅
+1 1/31
+1 1/29,1/31
+1 1/11,1/29,1/31
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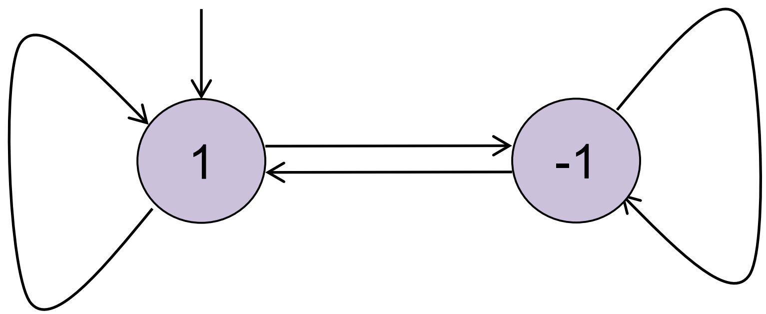

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Value of strongly-connected game is 0 independent of initial budgets.

Optimal strategy for player Min:
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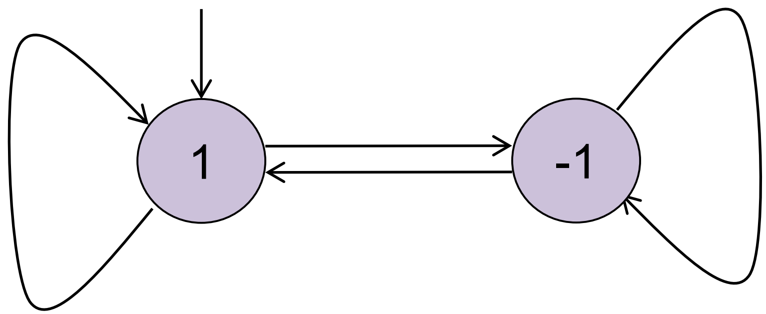
if M is empty, bid 0.

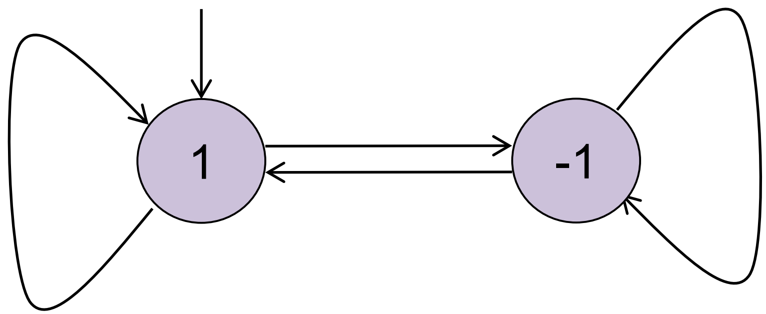
Either M infinitely often empty, or size of M bounded.

Poorman "Bowtie"



Initial budget ratio 1/2 for player Min:
optimal Min strategy same as before; then val ≤ 0.

Poorman "Bowtie"



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Initial budget ratio 2/3 for player Min:
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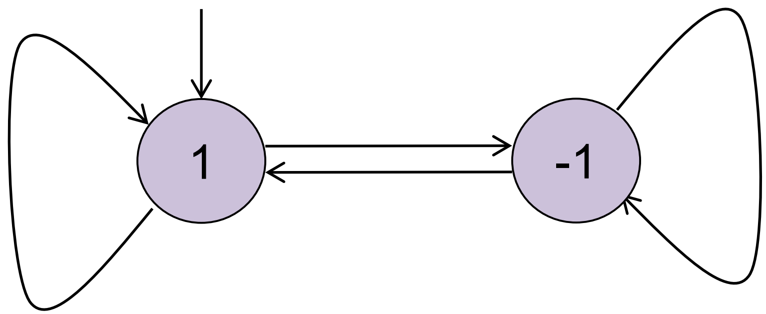

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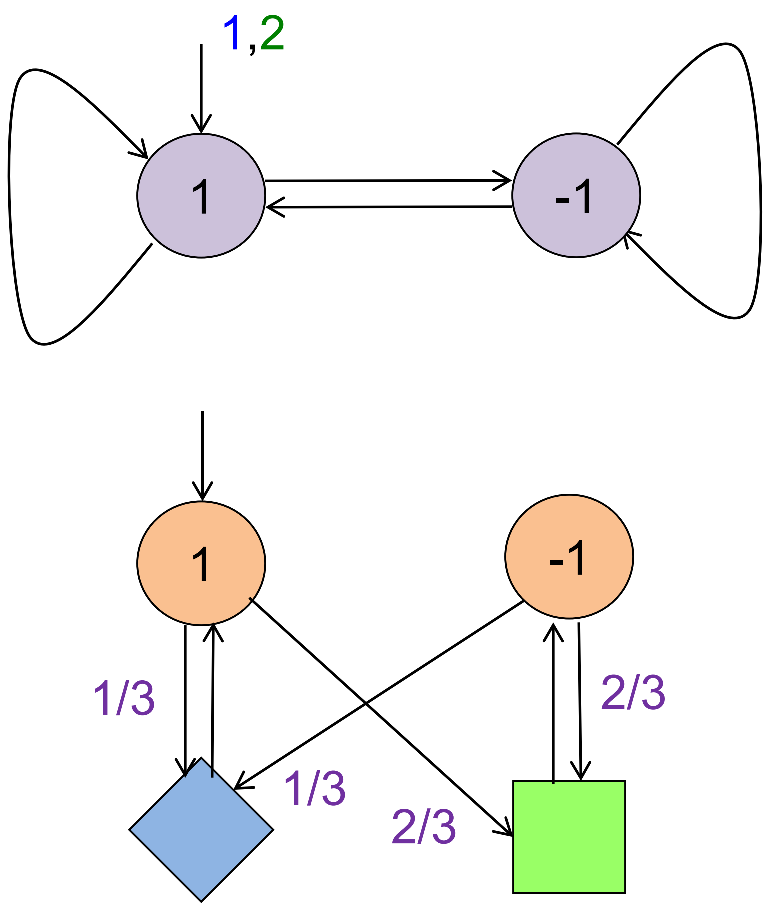
Value of strongly-connected game depends on initial budgets.

Equivalence with Biased Random-Turn Games



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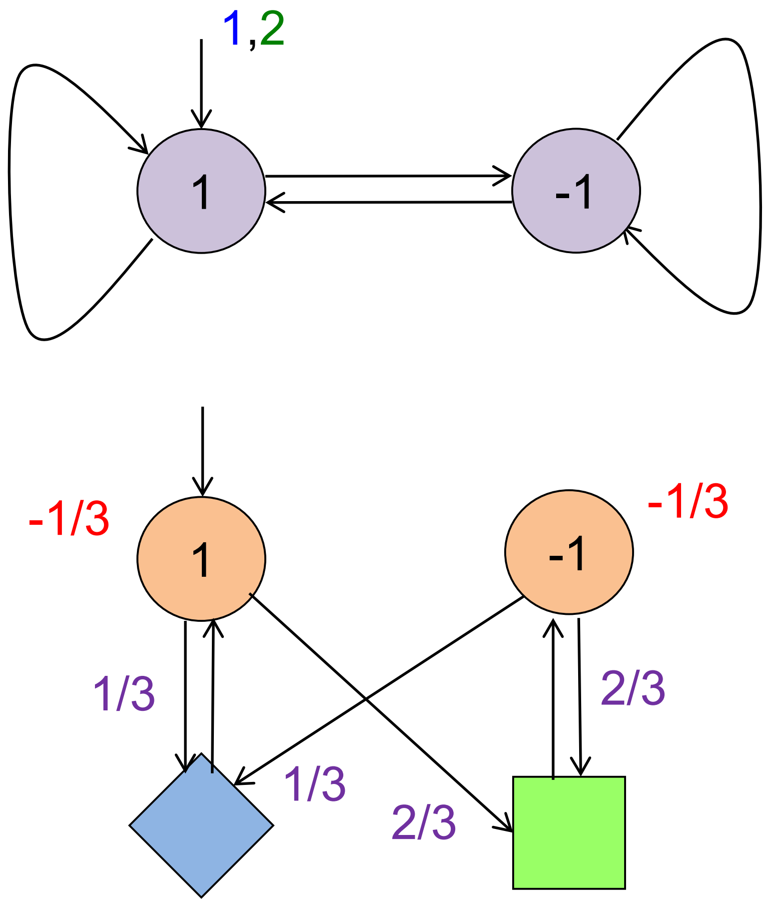

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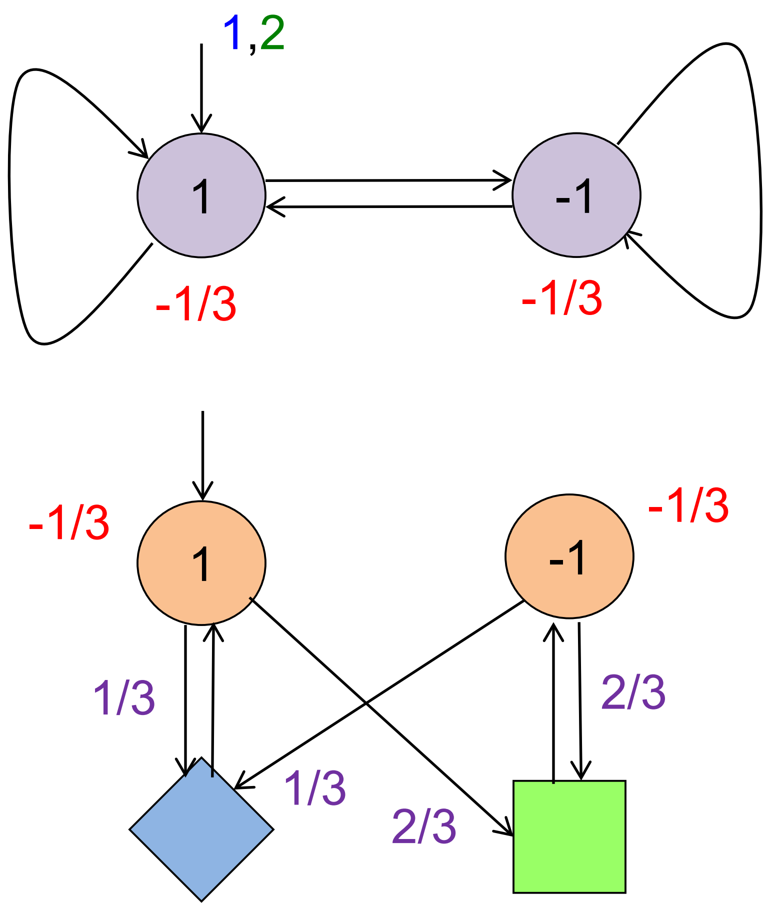
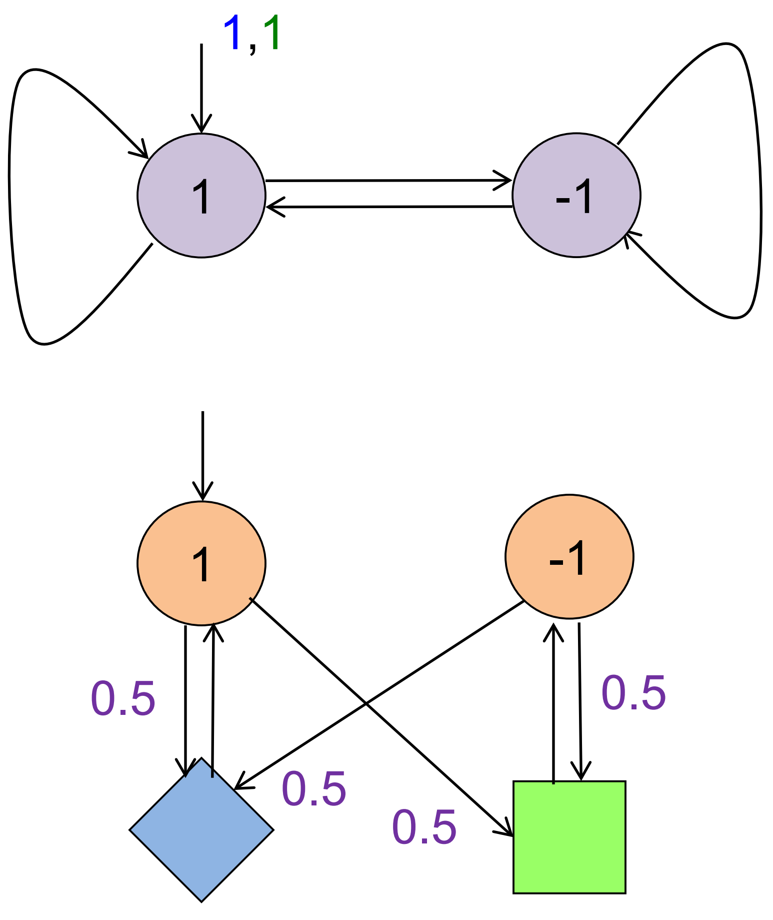
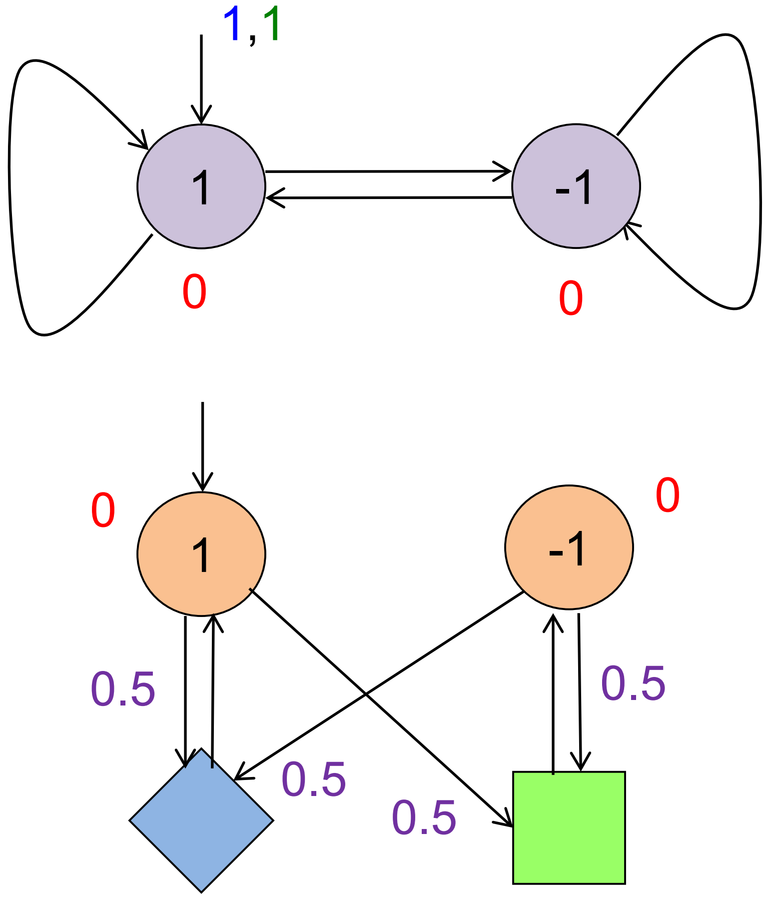


Equivalence with Biased Random-Turn Games

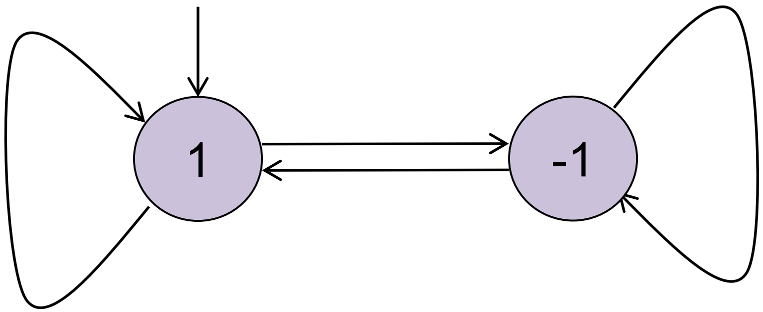


Equivalence with Biased Random-Turn Games



Richman = poorman with equal initial budgets

Which bidding modes do the players prefer?



Player with larger initial budget prefers poorman;
player with smaller initial budget prefers Richman.

Equal initial budgets: both modes are equivalent.

Much left to do

“Inverse” problems:
What is the threshold budget for reaching a target with a given probability?
What is the threshold budget for achieving a given mean payoff?

“Single-currency” problems:
What happens if payoffs and budgets are linked
(i.e., budgets can be recharged)?

“Non-zerosum” problems:
What happens if players have objectives that are not dual?
If there are more than 2 players?

Discrete Bids

If budgets and bids are integers (rather than reals), bidding games are a special case of concurrent games.

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If budgets and bids are integers (rather than reals), bidding games are a special case of concurrent games.

Tie-breaking becomes critical.

Some tie-breaking mechanisms ensure determinacy (e.g., round-robin, fair coin); others don't (e.g., whether or not the first bidding results in a tie determines which player wins all ties).

References

Richman bidding: Avni, H, Chonev; JACM 2019
Poorman bidding: Avni, H, Ibsen-Jensen; WINE 2018
Taxman bidding: Avni, H, Zikelic; MFCS 2019
Discrete bidding: Aghajohari, Avni, H; CONCUR 2019
All-pay bidding: Avni, Ibsen-Jensen, Tkadlec; AAAI 2020

Thank you!