

# Finite Automata over Conway Semirings

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A starsemiring satisfying the sum – star – equation and the product – star – equation is called Conway semiring. Each complete starsemiring is a Conway semiring. The semirings of  $n \times n$  – matrices with entries in a Conway semiring and of power series with coefficients in a Conway semiring are again Conway semirings.

Let  $S$  be a Conway semiring and  $S'$  be a subset of  $S$  containing 0 and 1. A finite  $S'$  – automaton with state set  $\{1, \dots, n\}$  is defined by an transition matrix  $M$ , an initial row vector  $I$  and a final column vector  $P$ , all of dimension  $n$  with entries in  $S'$ . Its behavior is given by  $IM^*P$ .

Let  $\text{Rec}(S')$  be the collection of the behaviors of all finite  $S'$  – automata. Denote by  $\text{Rat}(S')$  the least starsemiring closed under the rational operations  $+$ ,  $\cdot$ ,  $*$  and containing  $S'$ . Then we prove the Kleene theorem  $\text{Rec}(S') = \text{Rat}(S')$ .

If the basic semiring is a power series semiring with coefficients in a Conway semiring, we prove a Kleene – Schützenberger theorem.