Strange Parameterized Complexity Results of Natural Combinatorial Problems in Automata Theory and Algebra

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Downey and Fellows pioneered in the 1990s the theory of Parameterized Complexity. This theory can be seen as a refinement of the classical computational complexity theory that talks about complexity classes like NP or PSPACE. The idea is to add a secondary measurement of the 'size' of an instance by a parameterization function κ. Assuming that κ-values of instance x are small, also NP-hard problems might be efficiently solvable, in the sense of allowing for algorithms solving x in time $f(\kappa(x)) \cdot |x|^{\mathcal{O}(1)}$, yielding the complexity class FPT. This has led to the discussion of the so-called W-hierarchy, later complemented by Flum and Grohe with the so-called A-hierarchy:

- $\mathsf{FPT} \subseteq \mathsf{W}[1] \subseteq \mathsf{W}[2] \subseteq \mathsf{W}[3] \subseteq \cdots \subseteq \mathsf{W}[\mathsf{SAT}] \subseteq \mathsf{W}[\mathsf{P}] \subseteq (\mathsf{para\text{-}NP} \cap \mathsf{XP})$;
- $\mathsf{FPT} \subseteq \mathsf{W}[1] = \mathsf{A}[1] \subseteq \mathsf{W}[2] \subseteq \mathsf{A}[2] \subseteq \mathsf{A}[3] \subseteq \cdots \subseteq \mathsf{AW}[\mathsf{P}] \subseteq \mathsf{XP}$.

Moreover, in general $\mathsf{W}[t] \subseteq \mathsf{A}[t]$ is known. For many computational problems and even more parameterization functions, it has become clear where to place them within these hierarchies. However, most research focused so far on problems from graph theory and logic. Automata theory and also algebra has been (so far) mostly neglected. As we will explain in this talk, as both areas yield interesting parameterized complexity results, motivating a focus on possibly new complexity classes. In particular, we will champion a class that we call $\mathsf{W}[\mathsf{Sync}]$, and we prove that

$$\mathsf{W}[2] \subseteq \mathsf{W}[\mathsf{Sync}] \subseteq \mathsf{WNL} \cap \mathsf{W}\,[\mathsf{P}]\, \cap \mathsf{A}[2]\,.$$

Also, while in particular graph problems are mostly classified into FPT, $\mathsf{W}[1]$ or $\mathsf{W}[2]$, this is no longer the case for typical automata problems.